Quiz 6

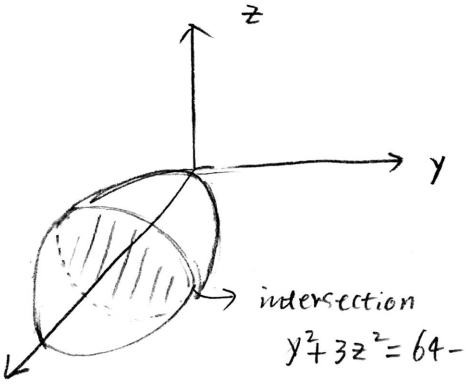
MATH 2162.02

RECITATION TIME: NAME:

There is another problem on the back page.

Problem 1. (5 points) Set up an integral expression that would compute the volume of the solid region between the paraboloids $x = y^2 + 3z^2$ and $x = 64 - 3y^2 - z^2$. You do not need to evaluate the integral.

Answer:

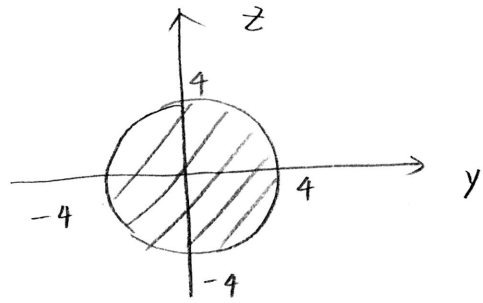


intersection

$y^2 + 3z^2 = 64 - 3y^2 - z^2$

$\Leftrightarrow \quad 4y^2 + 4z^2 = 64$

$y^2 + z^2 = 16$



$$\text{Volume} = \iint_{R_{yz}} 64 - 3y^2 - z^2 - (y^2 - 3z^2)\, dA$$

Method I: Cartesian

$$V = \int_{-4}^{4} \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} 64 - 4y^2 - 4z^2 \, dz\, dy$$

Method II: Polar

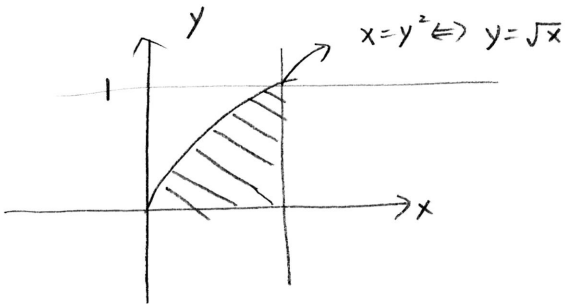
$$V = \int_0^{2\pi} \int_0^4 (64 - 4r^2)\, r\, dr\, d\theta$$

Problem 2. (5 points) Compute the double integral

$$\int_0^1 \int_{y^2}^1 4ye^{x^2}\, dx\, dy$$

Hint: You might want to change the order of integration.

Answer:



$$\int_0^1 \int_0^{\sqrt{x}} 4ye^{x^2}\, dy\, dx$$

$$= \int_0^1 2y^2 e^{x^2} \Big|_0^{\sqrt{x}} dx$$

$$= \int_0^1 2xe^{x^2} dx$$

$$= e^{x^2} \Big|_0^1$$

$$= e - 1$$