Math 1172 - Autumn 2015 Name: ______________

Quiz 3 - In Class Recitation Instructor: ______________

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [6 points] An arch is to be modeled by the function $y = e^{-2x} + \frac{1}{16}e^{2x}$ on the interval $[-1, 1]$. Find the length of the arch. You do not need to simplify your final answer.

$$y' = -2e^{-2x} + \frac{1}{16} \cdot 2e^{2x}$$
$$= \frac{1}{8}e^{2x} - 2e^{-2x}$$

$$(y')^2 = \left(\frac{1}{8}e^{2x}\right)^2 - 2 \cdot \frac{1}{8}e^{2x} \cdot 2e^{-2x} + \left(2e^{-2x}\right)^2$$
$$= \left(\frac{1}{8}e^{2x}\right)^2 - \frac{1}{2} + \left(2e^{-2x}\right)^2$$

$$1 + (y')^2 = \left(\frac{1}{8}e^{2x}\right)^2 + \frac{1}{2} + \left(2e^{-2x}\right)^2$$
$$= \left(\frac{1}{8}e^{2x}\right)^2 + 2 \cdot \frac{1}{8}e^{2x} \cdot 2e^{-2x} + \left(2e^{-2x}\right)^2$$
$$= \left(\frac{1}{8}e^{2x} + 2e^{-2x}\right)^2$$

So the length

$$L = \int_{-1}^{1} \sqrt{1 + (y'(x))^2}\, dx$$
$$= \int_{-1}^{1} \sqrt{\left(\frac{1}{8}e^{2x} + 2e^{-2x}\right)^2}\, dx$$
$$= \int_{-1}^{1} \frac{1}{8}e^{2x} + 2e^{-2x}\, dx$$
$$= \frac{1}{16}e^{2x} - e^{-2x} \Big|_{-1}^{1}$$
$$= \left(\frac{1}{16}e^{2} - e^{-2}\right) - \left(\frac{1}{16}e^{-2} - e^{2}\right)$$
$$= \frac{17}{16}e^{2} - \frac{17}{16}e^{-2} = \frac{17}{16}\left(e^{2} - e^{-2}\right)$$