Math 1172 - Autumn 2015 Name: _____

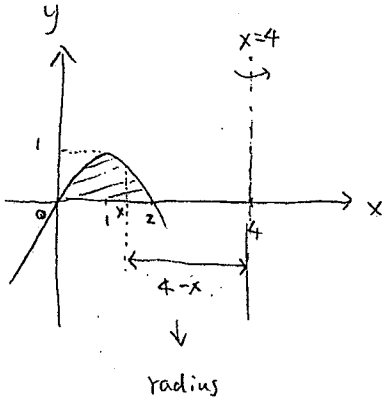
Quiz 2 - Take Home Recitation Instructor: _____

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [4 points] Consider the region R bounded by the graph $y = -x^2+2x$ and the x-axis.

$y = -x^2+2x = -(x^2-2x+1)+1 = -(x-1)^2+1$.

a) Find the volume of the solid of revolution formed by revolving the region R about the line $x = 4$.



shell method.

$$\text{Vol}\left(= \int_a^b 2\pi \cdot \text{radius} \cdot \text{height} \cdot dx\right).$$

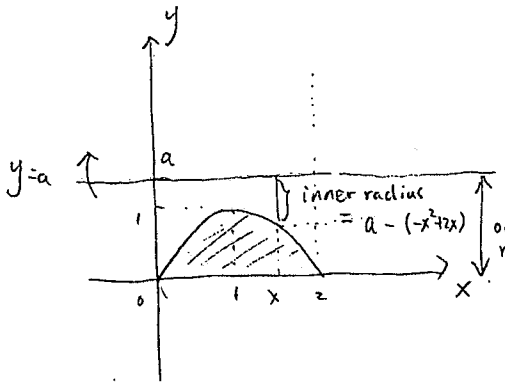
$$= \int_0^2 2\pi(4-x)(-x^2+2x)\,dx$$

$$= 2\pi \int_0^2 x^3 - 6x^2 + 8x \, dx$$

$$= 2\pi \left(\frac{x^4}{4} - 2x^3 + 4x^2\right)\Big|_0^2 = 2\pi\left(\frac{16}{4} - 2\cdot 8 + 4\cdot 4 - 0\right)$$

$$= 8\pi$$

b) Find the volume of the solid of revolution formed by revolving the region R about the line $y = a$ where $a > 1$ is a constant.



washer method.

$$\text{Vol}\left(= \int_a^b \pi\left[(\text{outer radius})^2 - (\text{inner radius})^2\right] dx\right)$$

$$= \int_0^2 \pi\left[a^2 - \left(a-(-x^2+2x)\right)^2\right] dx$$

$$= \pi \int_0^2 -x^4 + 4x^3 - (2a+4)x^2 + 4ax \, dx$$

$$= \pi\left(-\frac{x^5}{5} + x^4 - \frac{2a+4}{3}x^3 + 2ax^2\right)\Big|_0^2$$

$$= \pi\left(\frac{8a}{3} - \frac{16}{15}\right)$$

c) Find any values of a for which the volume of the solid in part (b) is the same as the volume of the solid in part (a). You may use a calculator and approximate any answers to two decimal places.

$$8\pi = \pi\left(\frac{8a}{3} - \frac{16}{15}\right) \iff 8 = \frac{8a}{3} - \frac{16}{15}$$

$$\iff 1 = \frac{a}{3} - \frac{2}{15}$$

$$\iff \frac{a}{3} = 1 + \frac{2}{15} \iff a = 3 + \frac{2}{5} = \frac{17}{5}$$