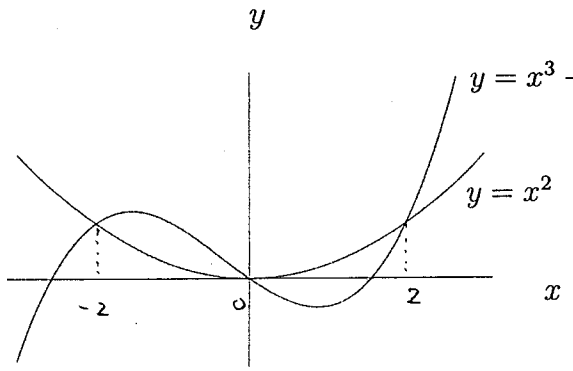
Math 1172 - Autumn 2015 Name: _____

Quiz 1 - In Class Recitation Instructor: _____

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [6 points] Find the area of the entire region that is bounded by the graphs of $y = x^2$ and $y = x^3 + x^2 - 4x$. Give the exact answer (you do not need to simplify the numerical value.)



① Find the intersections.

Set $x^3 + x^2 - 4x = x^2$.

So $0 = x^3 - 4x = x(x^2 - 4) = x(x-2)(x+2)$

The curves intersect when $x = 0, 2, -2$.

② Area $= \int_a^b (\text{upper} - \text{lower})\, dx$.

On $[-2, 0]$, $y = x^3 + x^2 - 4x$ is the upper curve;

On $[0, 2]$, $y = x^2$ is the upper curve.

Then the desired area is

$$\int_{-2}^{0} (x^3 + x^2 - 4x) - x^2\, dx + \int_0^2 x^2 - (x^3 + x^2 - 4x)\, dx$$

$$= \int_{-2}^{0} x^3 - 4x\, dx + \int_0^2 -x^3 + 4x\, dx$$

$$= \frac{x^4}{4} - 2x^2 \Big|_{-2}^{0} + \left(-\frac{x^4}{4} + 2x^2\right)\Big|_0^2$$

$$= 0 - \left(\frac{(-2)^4}{4} - 2(-2)^2\right) + \left(-\frac{2^4}{4} + 2 \cdot 2^2\right) - 0$$

$$= 8$$