Table 10-1: Properties of the Laplace transform.

Property	$f(t)$		$\mathbf{F}(\mathrm{s}) = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K\ f(t)$	$\longleftrightarrow$	$K\ \mathbf{F}(\mathrm{s})$
2. Linearity	$K_1\ f_1(t) + K_2\ f_2(t)$	$\longleftrightarrow$	$K_1\ \mathbf{F}_1(\mathrm{s}) + K_2\ \mathbf{F}_2(\mathrm{s})$
3. Time scaling	$f(at), \quad a > 0$	$\longleftrightarrow$	$\dfrac{1}{a}\ \mathbf{F}\left(\dfrac{\mathrm{s}}{a}\right)$
4. Time shift	$f(t-T)\ u(t-T)$	$\longleftrightarrow$	$e^{-T\mathrm{s}}\ \mathbf{F}(\mathrm{s})$
5. Frequency shift	$e^{-at}\ f(t)$	$\longleftrightarrow$	$\mathbf{F}(\mathrm{s}+a)$
6. Time 1st derivative	$f' = \dfrac{df}{dt}$	$\longleftrightarrow$	$\mathrm{s}\ \mathbf{F}(\mathrm{s}) - f(0^-)$
7. Time 2nd derivative	$f'' = \dfrac{d^2 f}{dt^2}$	$\longleftrightarrow$	$\mathrm{s}^2\mathbf{F}(\mathrm{s}) - \mathrm{s}f(0^-) - f'(0^-)$
8. Time integral	$\displaystyle\int_0^t f(t)\ dt$	$\longleftrightarrow$	$\dfrac{1}{\mathrm{s}}\ \mathbf{F}(\mathrm{s})$
9. Frequency derivative	$t\ f(t)$	$\longleftrightarrow$	$-\dfrac{d}{d\mathrm{s}}\ \mathbf{F}(\mathrm{s}) = -\mathbf{F}'(\mathrm{s})$
10. Frequency integral	$\dfrac{f(t)}{t}$	$\longleftrightarrow$	$\displaystyle\int_{\mathrm{s}}^{\infty} \mathbf{F}(\mathrm{s})\ d\mathrm{s}$
11. Initial value	$f(0^+)$	$=$	$\displaystyle\lim_{\mathrm{s}\to\infty} \mathrm{s}\ \mathbf{F}(\mathrm{s})$
12. Final value	$f(\infty)$	$=$	$\displaystyle\lim_{\mathrm{s}\to 0} \mathrm{s}\ \mathbf{F}(\mathrm{s})$
13. Convolution	$f_1(t) * f_2(t)$	$\longleftrightarrow$	$\mathbf{F}_1(\mathrm{s})\ \mathbf{F}_2(\mathrm{s})$

Table 10-2: Examples of Laplace transform pairs. Note that $f(t) = 0$ for $t < 0^-$.

Laplace Transform Pairs			
	$f(t)$		$\mathbf{F}(s) = \mathcal{L}[f(t)]$
1	$\delta(t)$	$\Leftrightarrow$	1
1a	$\delta(t-T)$	$\Leftrightarrow$	e^{-Ts}
2	$u(t)$	$\Leftrightarrow$	$\frac{1}{s}$
2a	$u(t-T)$	$\Leftrightarrow$	$\frac{e^{-Ts}}{s}$
3	$e^{-at}\,u(t)$	$\Leftrightarrow$	$\frac{1}{s+a}$
3a	$e^{-a(t-T)}\,u(t-T)$	$\Leftrightarrow$	$\frac{e^{-Ts}}{s+a}$
4	$t\,u(t)$	$\Leftrightarrow$	$\frac{1}{s^2}$
4a	$(t-T)\,u(t-T)$	$\Leftrightarrow$	$\frac{e^{-Ts}}{s^2}$
5	$t^2\,u(t)$	$\Leftrightarrow$	$\frac{2}{s^3}$
6	$te^{-at}\,u(t)$	$\Leftrightarrow$	$\frac{1}{(s+a)^2}$
7	$t^2e^{-at}\,u(t)$	$\Leftrightarrow$	$\frac{2}{(s+a)^3}$
8	$t^{n-1}e^{-at}\,u(t)$	$\Leftrightarrow$	$\frac{(n-1)!}{(s+a)^n}$
9	$\sin\omega t\,u(t)$	$\Leftrightarrow$	$\frac{\omega}{s^2+\omega^2}$
10	$\sin(\omega t+\theta)\,u(t)$	$\Leftrightarrow$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
11	$\cos\omega t\,u(t)$	$\Leftrightarrow$	$\frac{s}{s^2+\omega^2}$
12	$\cos(\omega t+\theta)\,u(t)$	$\Leftrightarrow$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
13	$e^{-at}\sin\omega t\,u(t)$	$\Leftrightarrow$	$\frac{\omega}{(s+a)^2+\omega^2}$
14	$e^{-at}\cos\omega t\,u(t)$	$\Leftrightarrow$	$\frac{s+a}{(s+a)^2+\omega^2}$
15	$2e^{-at}\cos(bt-\theta)\,u(t)$	$\Leftrightarrow$	$\frac{e^{j\theta}}{s+a+jb}+\frac{e^{-j\theta}}{s+a-jb}$
16	$\frac{2t^{n-1}}{(n-1)!}\,e^{-at}\cos(bt-\theta)\,u(t)$	$\Leftrightarrow$	$\frac{e^{j\theta}}{(s+a+jb)^n}+\frac{e^{-j\theta}}{(s+a-jb)^n}$

4. Repeated Complex Poles:

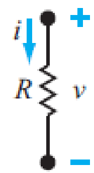
Same procedure as for repeated real poles

Table 10-3: Transform pairs for four types of poles.

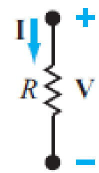
Pole	$\mathbf{F}(\mathbf{s})$	$f(t)$
1. Distinct real	$\dfrac{A}{\mathbf{s}+a}$	$Ae^{-at}\,u(t)$
2. Repeated real	$\dfrac{A}{(\mathbf{s}+a)^n}$	$A\,\dfrac{t^{n-1}}{(n-1)!}\,e^{-at}\,u(t)$
3. Distinct complex	$\left[\dfrac{Ae^{j\theta}}{\mathbf{s}+a+jb}+\dfrac{Ae^{-j\theta}}{\mathbf{s}+a-jb}\right]$	$2Ae^{-at}\cos(bt-\theta)\,u(t)$
4. Repeated complex	$\left[\dfrac{Ae^{j\theta}}{(\mathbf{s}+a+jb)^n}+\dfrac{Ae^{-j\theta}}{(\mathbf{s}+a-jb)^n}\right]$	$\dfrac{2At^{n-1}}{(n-1)!}\,e^{-at}\cos(bt-\theta)\,u(t)$

Table 10-4: Circuit models for R, L, and C in the s-domain.

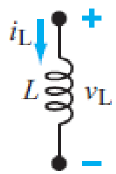
<table>
<tr><th>Time-Domain</th><th colspan="3">s-Domain</th></tr>
<tr><td>Resistor



$v = Ri$</td><td colspan="3">

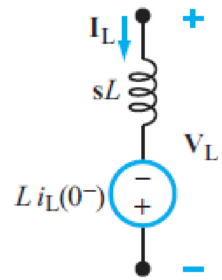


$\mathbf{V} = R\mathbf{I}$</td></tr>
<tr><td>Inductor

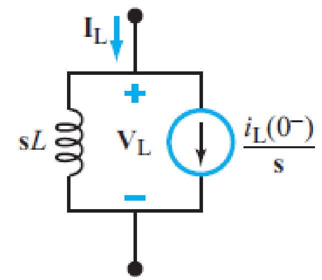


$v_L = L\,\dfrac{di_L}{dt}$

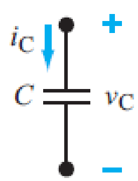
$i_L = \dfrac{1}{L}\displaystyle\int_{0^-}^{t} v_L\,dt + i_L(0^-)$</td><td>



$\mathbf{V}_L = \mathbf{s}L\mathbf{I}_L - L\,i_L(0^-)$</td><td>OR</td><td>

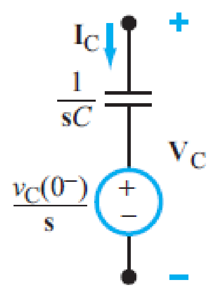


$\mathbf{I}_L = \dfrac{\mathbf{V}_L}{\mathbf{s}L} + \dfrac{i_L(0^-)}{\mathbf{s}}$</td></tr>
<tr><td>Capacitor

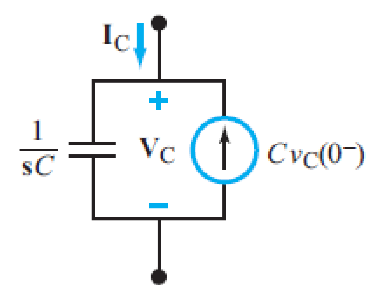


$i_C = C\,\dfrac{dv_C}{dt}$

$v_C = \dfrac{1}{C}\displaystyle\int_{0^-}^{t} i_C\,dt + v_C(0^-)$</td><td>



$\mathbf{V}_C = \dfrac{\mathbf{I}_C}{\mathbf{s}C} + \dfrac{v_C(0^-)}{\mathbf{s}}$</td><td>OR</td><td>



$\mathbf{I}_C = \mathbf{s}C\mathbf{V}_C - C\,v_C(0^-)$</td></tr>
</table>