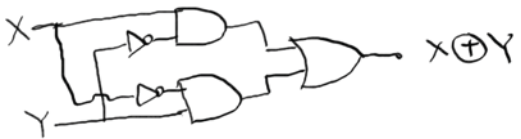
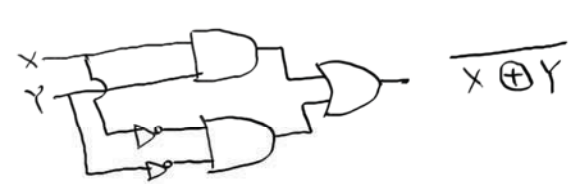


Lecture 9b

Exclusive - OR Operators

$$XOR \rightarrow X \oplus Y = X\bar{Y} + \bar{X}Y$$


A logic circuit diagram for an XOR gate. It consists of two 2-input AND gates followed by a 2-input OR gate. The first AND gate has inputs X and \bar{Y} (Y with a bubble). The second AND gate has inputs \bar{X} (X with a bubble) and Y. The outputs of the two AND gates are connected to the inputs of the OR gate, which produces the output $X \oplus Y$.

$$XNOR \rightarrow \overline{X \oplus Y} = XY + \bar{X}\bar{Y}$$


A logic circuit diagram for an XNOR gate. It consists of two 2-input AND gates followed by a 2-input OR gate, with an additional NOT gate at the output. The first AND gate has inputs X and Y. The second AND gate has inputs \bar{X} (X with a bubble) and \bar{Y} (Y with a bubble). The outputs of the two AND gates are connected to the inputs of the OR gate. The output of the OR gate is connected to a NOT gate, which produces the final output $\overline{X \oplus Y}$.

$$X \oplus 0 = X$$

$$X \oplus 1 = \bar{X}$$

$$X \oplus X = 0$$

$$X \oplus \bar{X} = 1$$

$$X \oplus \bar{Y} = \overline{X \oplus Y}$$

$$\bar{X} \oplus Y = \overline{X \oplus Y}$$

$$A \oplus B = B \oplus A$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$$

Odd Function

$$X \oplus Y \oplus Z$$

Hand-drawn Karnaugh map for a 4-variable function with variables X, Y, Z, and W. The map is a 2x4 grid. The columns are labeled with YZ (00, 01, 11, 10) and the rows are labeled with X (0, 1). The cells contain the following values: (0, 00) is 000, (0, 01) is 100, (0, 11) is 011, (0, 10) is 010, (1, 00) is 100, (1, 01) is 101, (1, 11) is 111, and (1, 10) is 110. Red annotations include a bracket above the columns labeled 'Y', a bracket below the rows labeled 'Z', and a bracket to the left of the rows labeled 'X'.

odd function

$$= \begin{cases} 1 & \text{when } x_j \text{ of } I's \text{ is} \\ 0 & " " " " " \end{cases}$$

odd

even



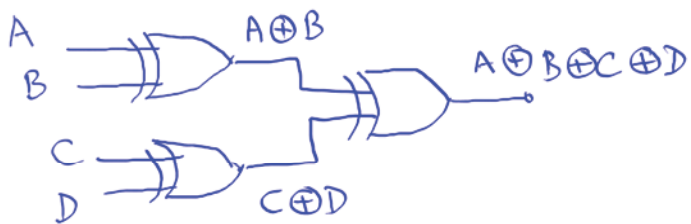
$$A \oplus B \oplus C \oplus D$$

CD 00 01 11 10

AB

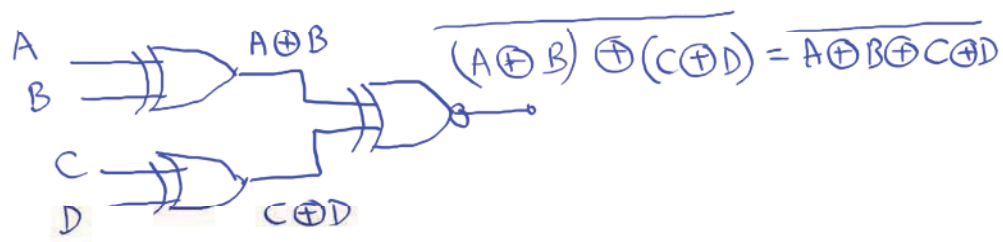
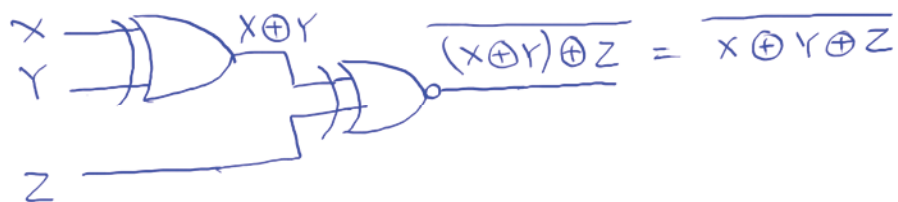
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

Odd function



Even function

$$\overline{X \oplus Y \oplus Z}$$



even function

$$= \begin{cases} 1 & \text{when no. of 1's is even} \\ 0 & \text{" " " " " odd} \end{cases}$$

Example: Design \rightarrow BCD-to-Excess-3 Code Converter

Specification:

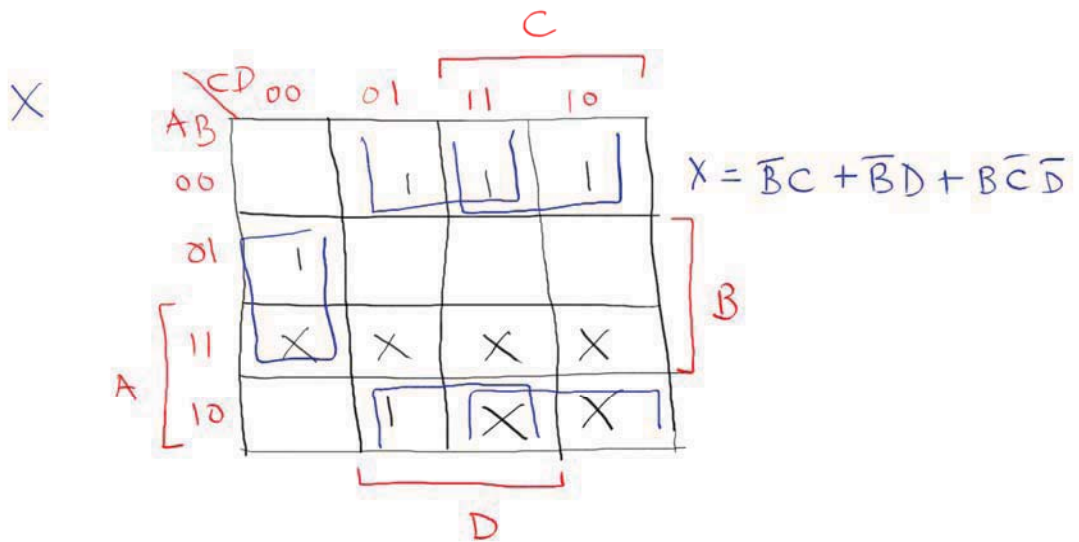
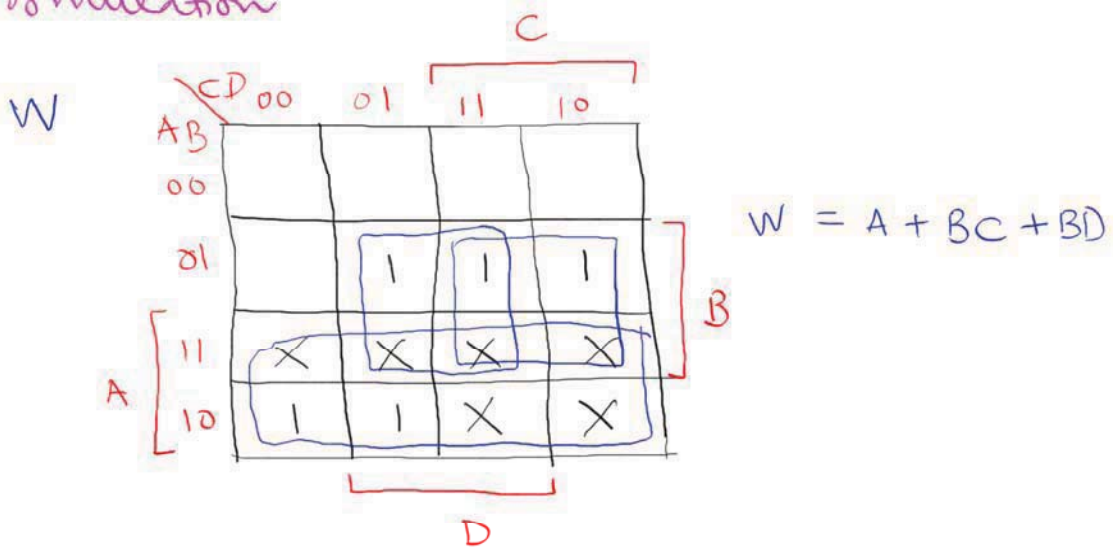
excess-3 code \rightarrow binary combination corresponding to the decimal digit plus 3

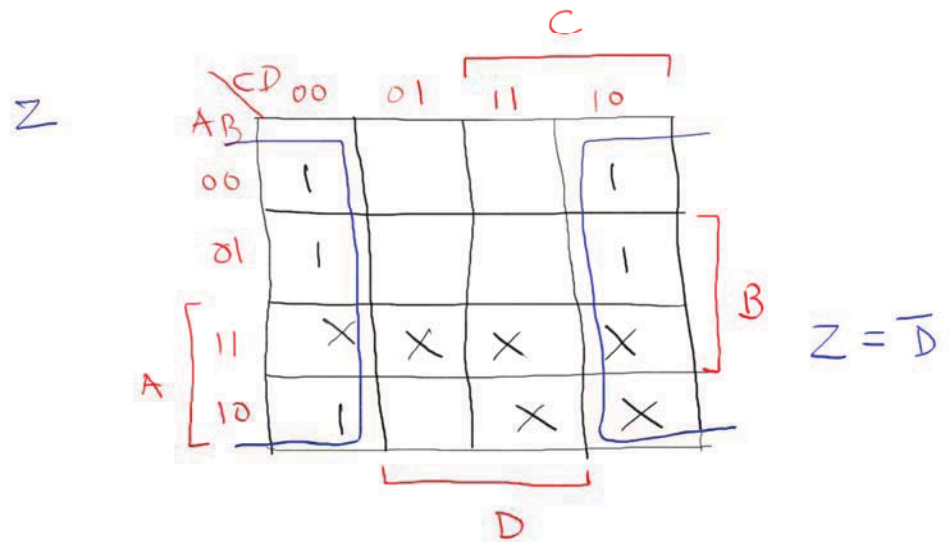
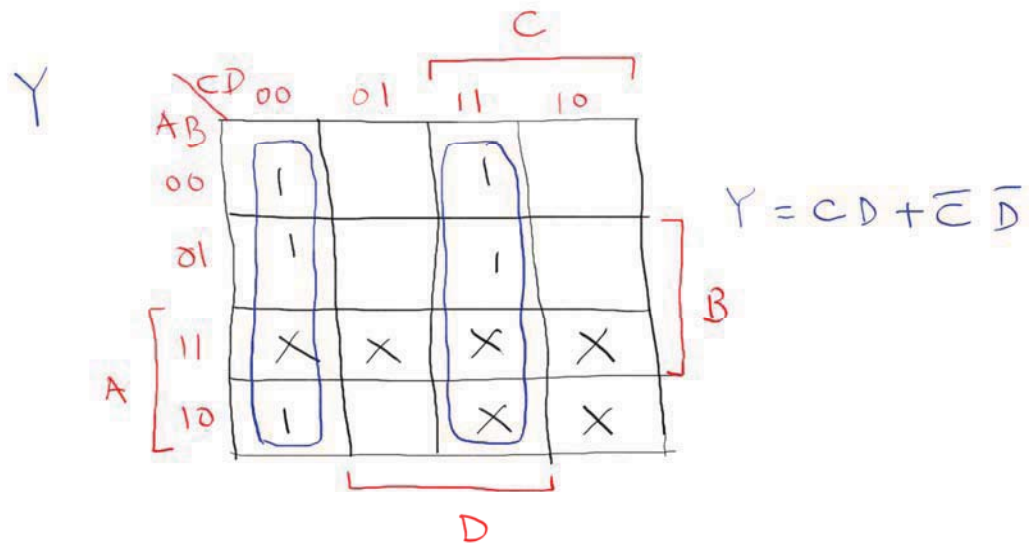
decimal digit \rightarrow 5 \rightarrow $5 + 3 = 8 \rightarrow 1000$ \leftarrow excess-3 code
9 \rightarrow $9 + 3 = 12 \rightarrow 1100$ \leftarrow

Decimal Digit	input BCD				Output Excess-3			
	A	B	C	D	W	X	Y	Z

0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	0	0	0
2	0	0	1	0	0	1	0	0
3	0	0	0	1	0	1	0	1
4	0	1	0	0	0	1	0	1
5	0	1	0	1	0	0	0	0
6	0	1	1	1	0	1	0	1
7	0	1	1	0	0	1	0	0
8	1	0	0	0	0	1	0	1
9	1	0	0	0	1	1	0	0

Formulation





$$\begin{aligned}
 W &= A + BC + BD \\
 X &= \overline{B}C + \overline{B}D + B\overline{C}\overline{D} \\
 Y &= CD + \overline{C}\overline{D} \\
 Z &= \overline{D}
 \end{aligned}
 \left. \vphantom{\begin{aligned} W \\ X \\ Y \\ Z \end{aligned}} \right\} \begin{array}{l} \text{2-level} \\ \text{implementation} \end{array}$$

↓ more simplification

$$T_1 = C + D$$

$$W = A + BC + BD = A + BT_1$$

$$X = \overline{B}C + \overline{B}D + B\overline{C}\overline{D} = \overline{B}T_1 + B\overline{C}\overline{D}$$

$$Y = CD + \overline{C}\overline{D}$$

$$Z = \overline{D}$$

