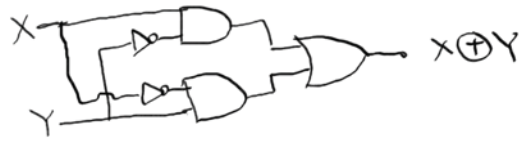
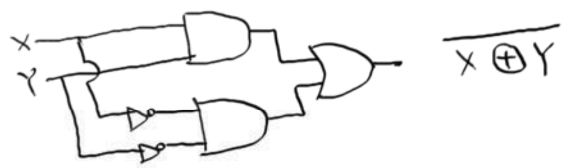
Lecture 9b

Exclusive-OR Operators

XOR $\rightarrow$ $X \oplus Y = X\bar{Y} + \bar{X}Y$



XNOR $\rightarrow$ $\overline{X \oplus Y} = XY + \bar{X}\bar{Y}$



$X \oplus 0 = X$ $\qquad$ $X \oplus 1 = \bar{X}$

$X \oplus X = 0$ $\qquad$ $X \oplus \bar{X} = 1$

$X \oplus \bar{Y} = \overline{X \oplus Y}$ $\qquad$ $\bar{X} \oplus Y = \overline{X \oplus Y}$

$A \oplus B = B \oplus A$

$(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$

Odd Function

$X \oplus Y \oplus Z$

X \ YZ	00	01	11	10
0	000	1 001	011	1 010
1	1 100	101	1 111	110

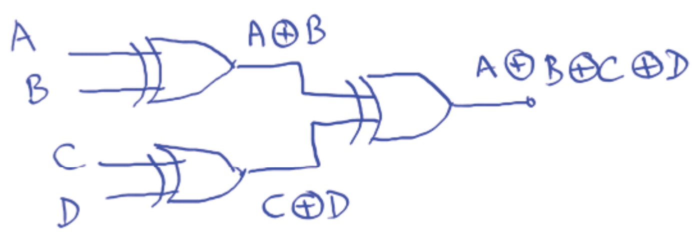
odd function

$= \begin{cases} 1 & \text{when \# of 1's is odd} \\ 0 & \text{" " " " even} \end{cases}$

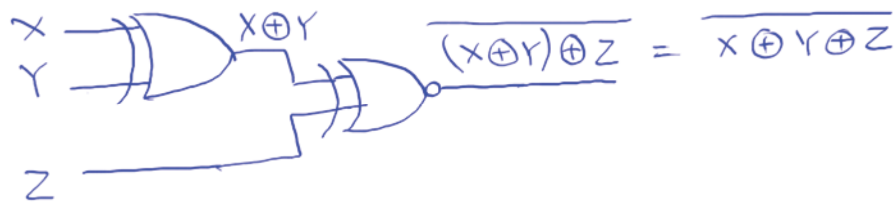
$A \oplus B \oplus C \oplus D$

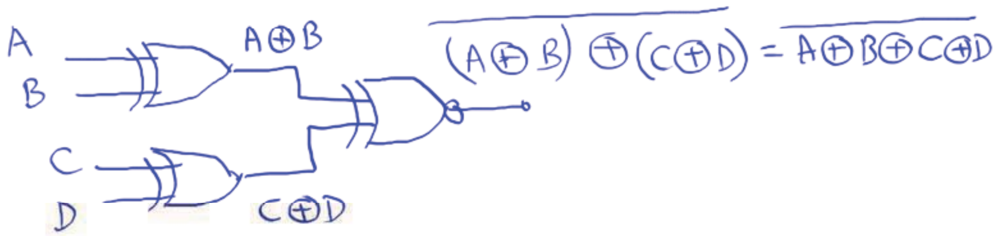
AB \ CD	00	01	11	10
00	0000	1 0001	0011	1 0010
01	1 0100	0101	1 0111	0110
11	1100	1 1101	1111	1 1110
10	1 1000	1001	1 1011	1010

Odd function



Even function $\overline{X \oplus Y \oplus Z}$





even function

$$= \begin{cases} 1 & \text{when \# of 1's is even} \\ 0 & \text{'' '' '' '' '' odd} \end{cases}$$

Example: Design → BCD-to-Excess-3 Code Converter

specification.

excess-3 code → binary combination corresponding to the decimal digit plus 3

decimal digit

5 → 5 + 3 = 8 → 1000 ← excess-3 code

9 → 9 + 3 = 12 → 1100

Decimal Digit	input BCD				Output Excess-3			
	A	B	C	D	W	X	Y	Z

0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

Formulation

W

AB \ CD	00	01	11	10
00				
01		1	1	1
11	X	X	X	X
10	1	1	X	X

$W = A + BC + BD$

X

AB \ CD	00	01	11	10
00		1	1	1
01	1			
11	X	X	X	X
10		1	X	X

$X = \bar{B}C + \bar{B}D + B\bar{C}\bar{D}$

Y

AB \ CD	00	01	11	10
00	1		1	
01	1		1	
11	X	X	X	X
10	1		X	X

C, D, A, B

$Y = CD + \bar{C}\bar{D}$

Z

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	X	X	X	X
10	1		X	X

C, D, A, B

$Z = \bar{D}$

$$\left.\begin{aligned} W &= A + BC + BD \\ X &= \bar{B}C + \bar{B}D + B\bar{C}\bar{D} \\ Y &= CD + \bar{C}\bar{D} \end{aligned}\right\} \text{2-level implementation}$$

$Z = \bar{D}$

$\downarrow$ more simplification

$T_1 = C + D$

$W = A + BC + BD = A + BT_1$

$$X = \bar{B}C + \bar{B}D + B\bar{C}\bar{D} = \bar{B}T_1 + B\bar{C}\bar{D}$$

$$Y = CD + \bar{C}\bar{D}$$

$$Z = \bar{D}$$