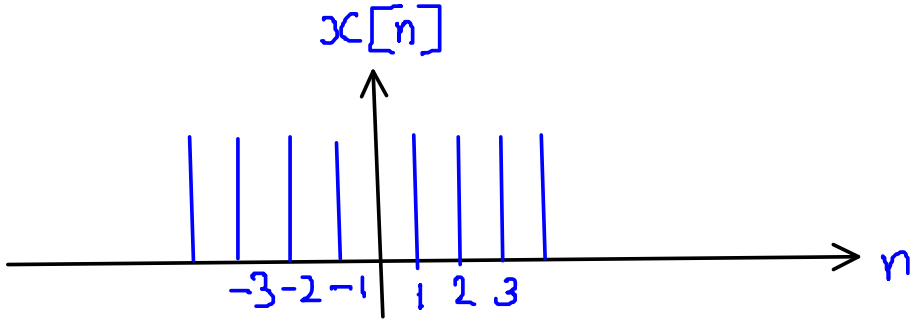
$$x[n] = A\cos(\hat{\omega}_0 n) = A\cos(2\pi \hat{f}_0 n)$$

$\hat{f}_0 = 0$

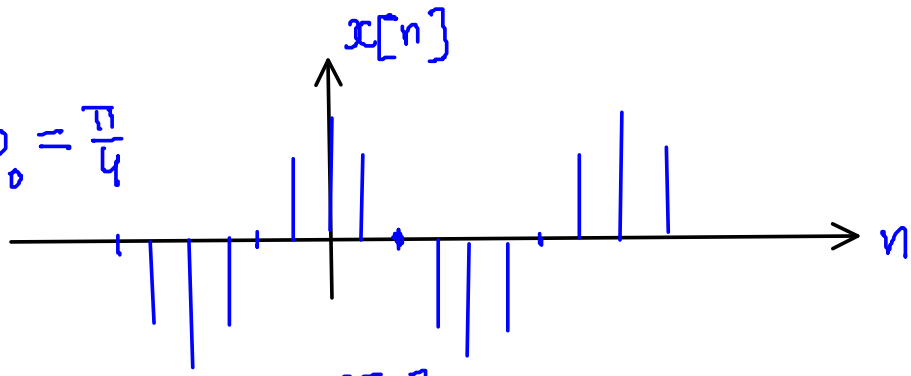


cycles/sample

$\hat{f}_0 = \frac{1}{8}$

8 samples per cycle

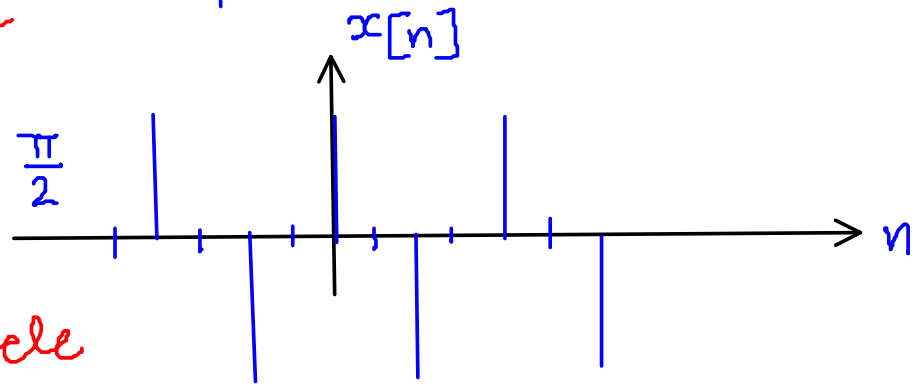
$\hat{\omega}_0 = \frac{\pi}{4}$



$\hat{f}_0 = \frac{1}{4}$

4 samples/cycle

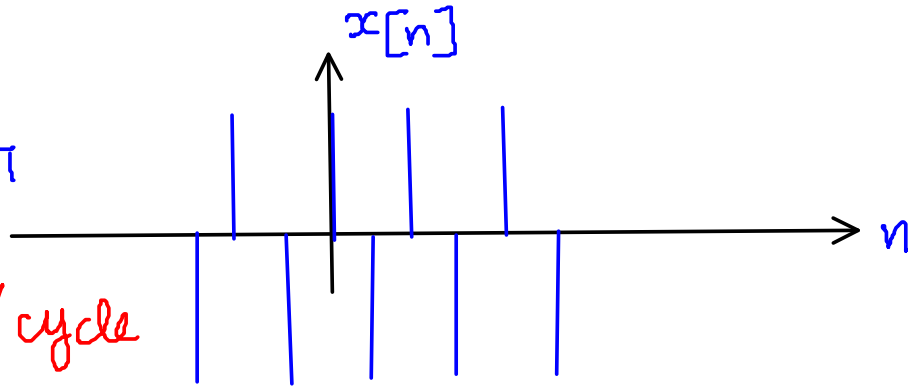
$\hat{\omega}_0 = \frac{\pi}{2}$



$\hat{f}_0 = \frac{1}{2}$

2 samples/cycle

$\hat{\omega}_0 = \pi$

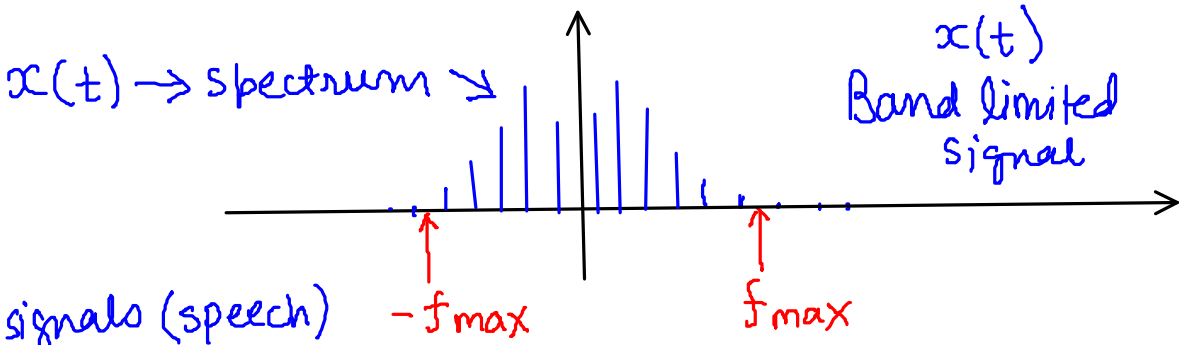


4-1.4 The Sampling Theorem

Fig 4-3 - How frequently must we sample in order to retain enough information to reconstruct the original continuous-time signal from the samples?

Shannon Sampling Theorem:

A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$ if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$



Audio signals (speech)
$f_{max} \sim 3000\,Hz$

TV signals
$f_{max} \sim 5\,MHz$

$$f_s \geq 2f_{max} \qquad f_{max} \leq \frac{f_s}{2}$$

Nyquist rate

CD's use $f_s = 44.1\,KHz$ $\qquad \frac{f_s}{2} = 22\,KHz$

20 KHz → threshold of human hearing and perception

$$f_{max} = 20\,KHz \leq \frac{f_s}{2} = 22\,KHz$$

Sinusoid signal:

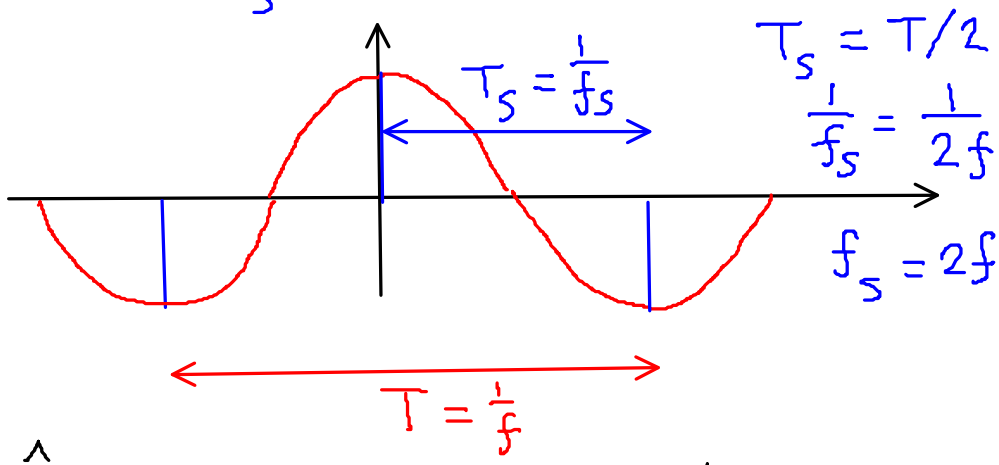
$$A\cos(2\pi f t + \phi)$$

$f_{max} = f$

$$f \leq \frac{f_s}{2} \qquad f_s \geq 2f$$

minimum sampling rate

$$f_s = 2f$$



$T_s = T/2$

$\frac{1}{f_s} = \frac{1}{2f}$

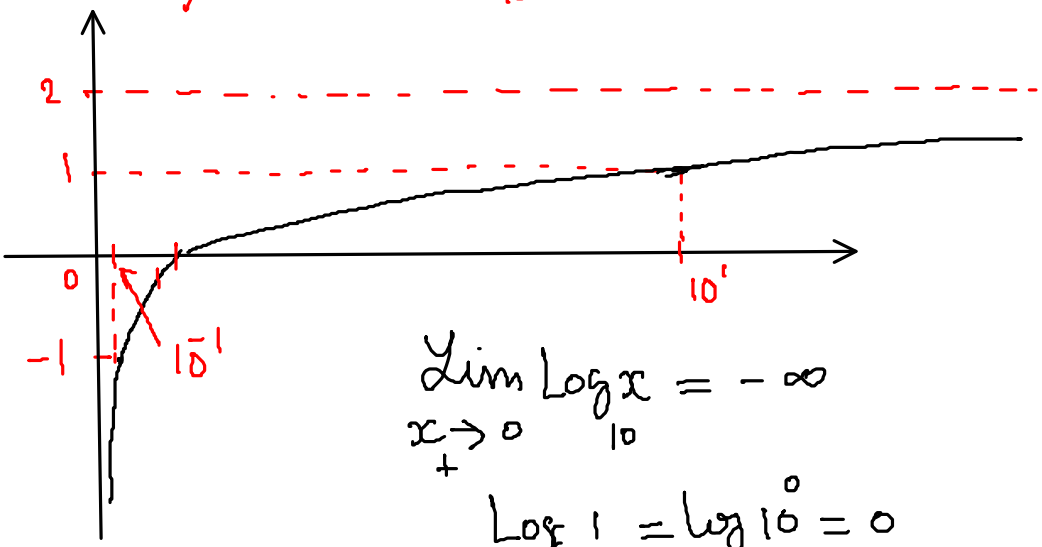
$f_s = 2f$

Note: $\hat{f}_0$ of discrete sinusoid $= \frac{1}{2}$

Decibels

$x(t) \rightarrow$ voltage or current signal

dB (Decibels) $\rightarrow 20 \log_{10} x(t)$

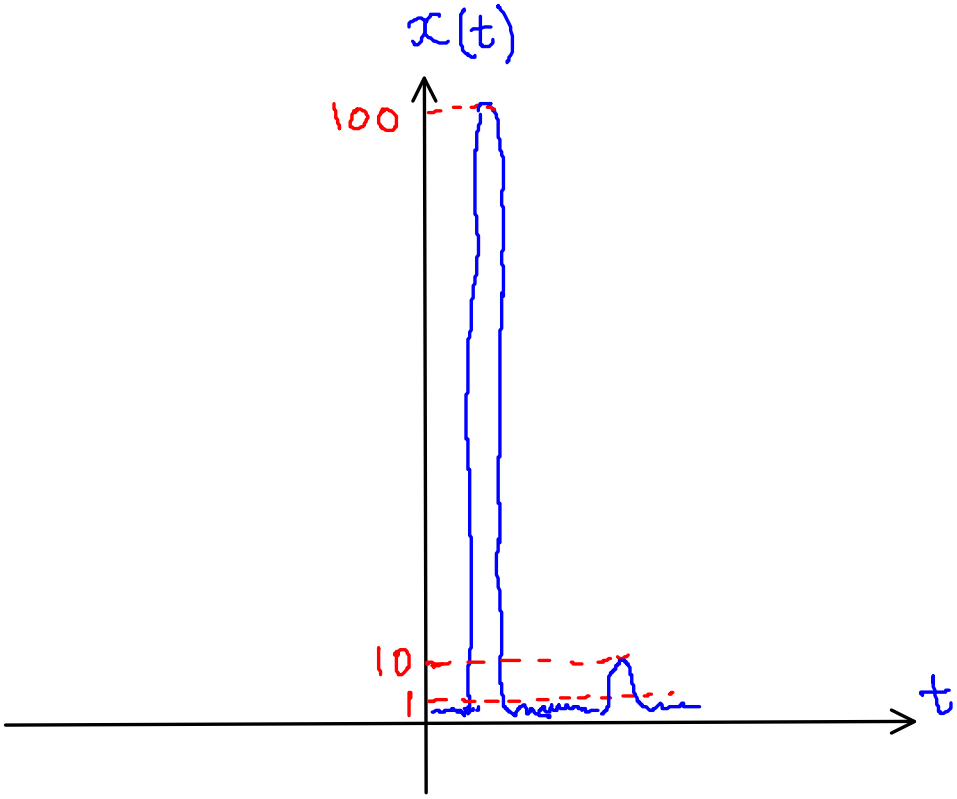


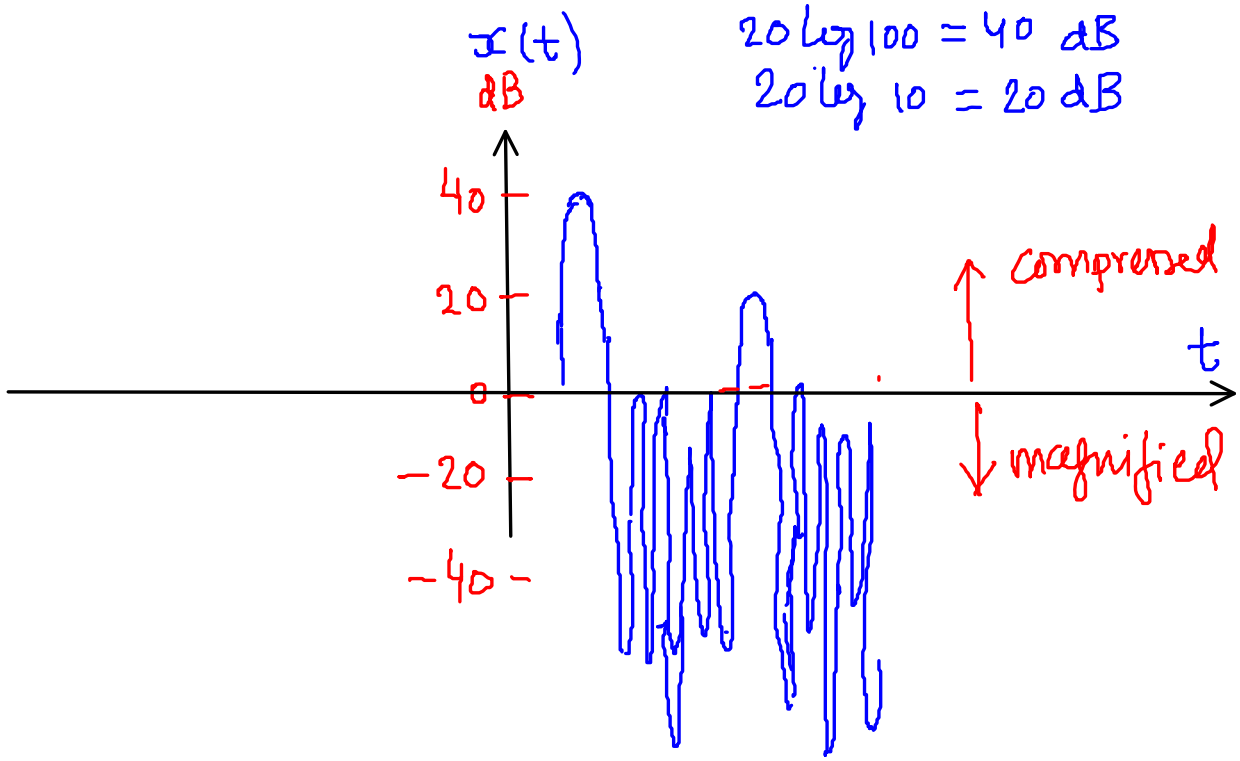
$$\lim_{x \to 0^+} \log_{10} x = -\infty$$

$$\log_{10} 1 = \log_{10} 10^0 = 0$$

$$\log_{10} 10^1 = 1$$

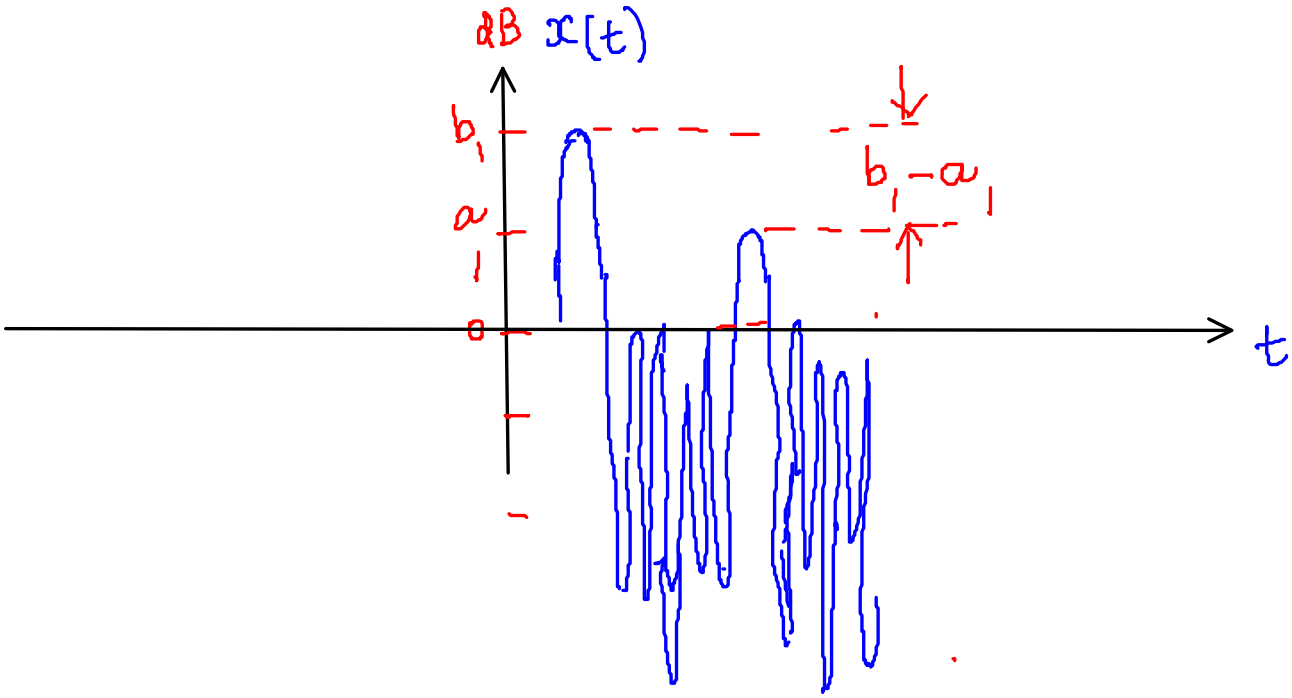
$$\log_{10} 10^2 = 2$$





$20 \log 100 = 40$ dB

$20 \log 10 = 20$ dB

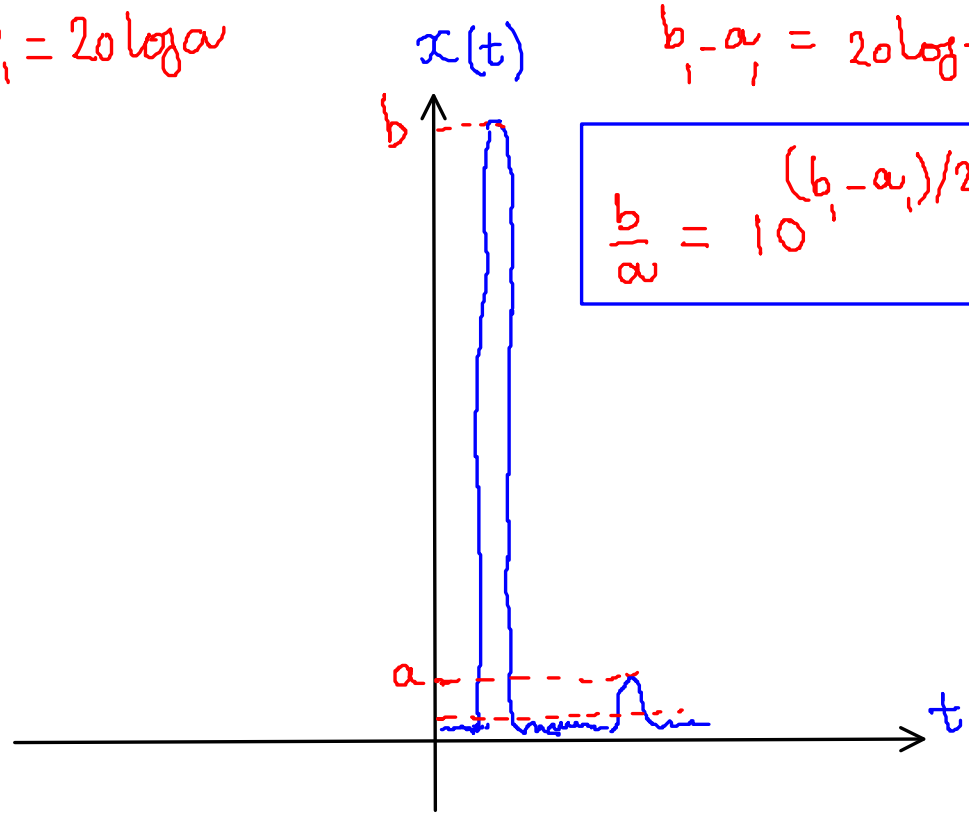


$b_1 = 20\log b$

$a_1 = 20\log a$

$b_1 - a_1 = 20\log b - 20\log a$

$b_1 - a_1 = 20\log\frac{b}{a}$



$$\boxed{\frac{b}{a} = 10^{(b_1 - a_1)/20}}$$