Division and multiplication by 2^m

Division: Unsigned Integers

Let a be an n-bit unsigned integer and let $m <= n-1$

$$a = a_{n-1} 2^{n-1} + \ldots + a_m 2^m + \ldots + a_0 2^0$$

Division by 2^m

$$a/2^m = [\ a_{n-1} 2^{n-1} + \ldots + a_m 2^m + \ldots + a_0 2^0]\ 2^{-m}$$

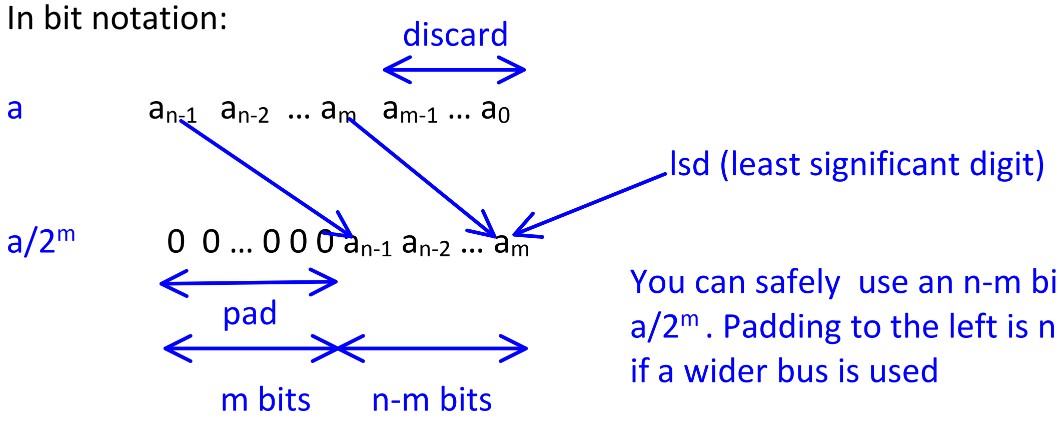
$$a/2^m = [\ \underbrace{a_{n-1} 2^{n-1-m} + \ldots + a_m 2^0}_{\text{Integer part}} \underbrace{+ \ldots + a_0 2^{-m}}_{\text{Fractional part}}]$$

Discard the fractional part

$$a/2^m = [\ a_{n-1} 2^{n-1-m} + \ldots + a_m 2^0\ ]$$

$a/2^m$ (after discarding the fractional part) is the same as shifting the bits of a by m bits to the right

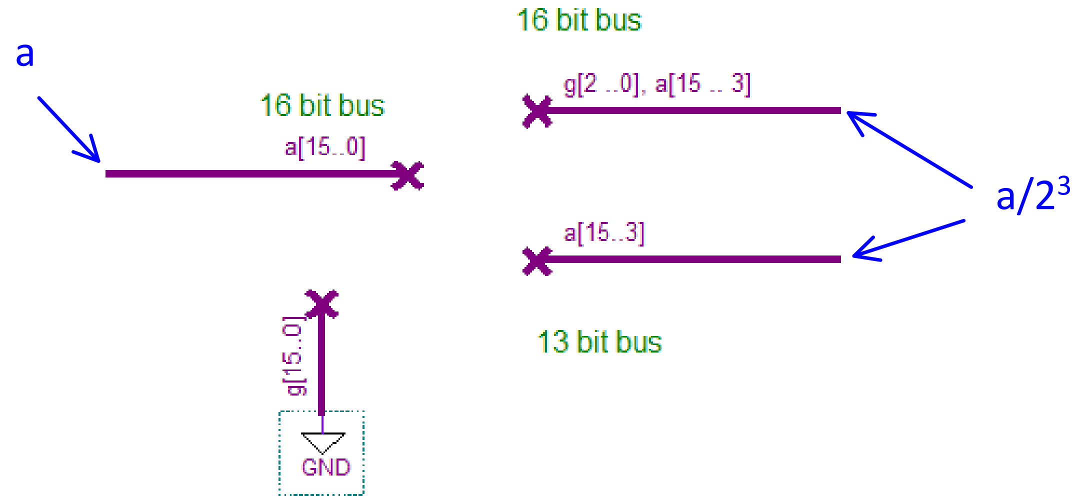
In bit notation:



You can safely use an n-m bit bus for $a/2^m$. Padding to the left is needed only if a wider bus is used

Division and multiplication by 2^m Page 2

Quartus example, unsigned division example: $n = 16 \quad m = 3$



Multiplication: Unsigned Integers

$a = a_{n-1} 2^{n-1} + \ldots + a_m 2^m + \ldots + a_0 2^0$

Multiplication by 2^m

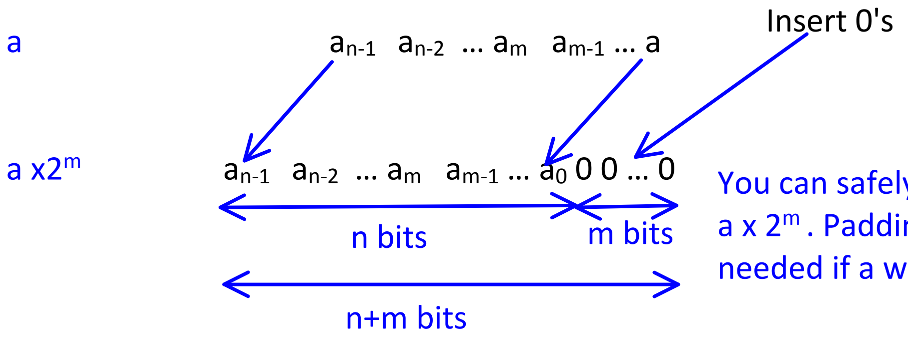
$a \times 2^m = [\, a_{n-1} 2^{n-1} + \ldots + a_0 2^0] \, 2^m$

$a \times 2^m = [\, a_{n-1} 2^{n+m-1} + \ldots + \ \ldots + a_0 2^m]$

$= [\, a_{n-1} 2^{n+m-1} + \ldots + \ \ldots + a_0 2^m + 0 \times 2^{m-1} + \ \ldots + 0 \times 2^0]$

a x 2m is the same as shifting all the bits of a by m bits to the left

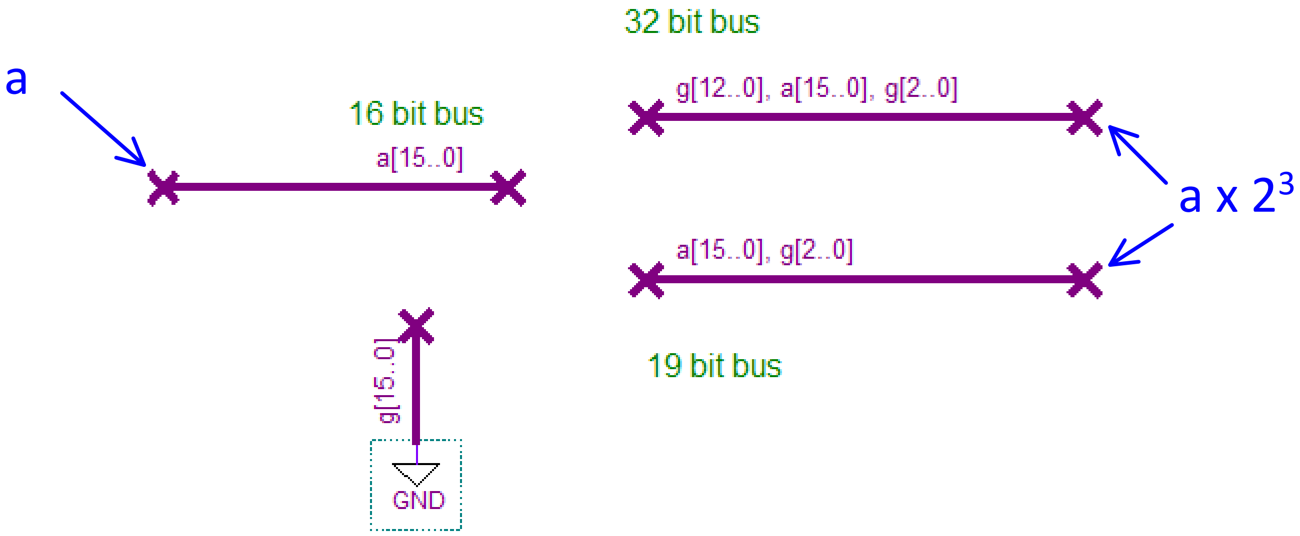
In bit notation:



You can safely use an n+m bit bus for $a \times 2^m$. Padding to the left by 0's is needed if a wider bus is used

Division and multiplication by 2^m Page 3

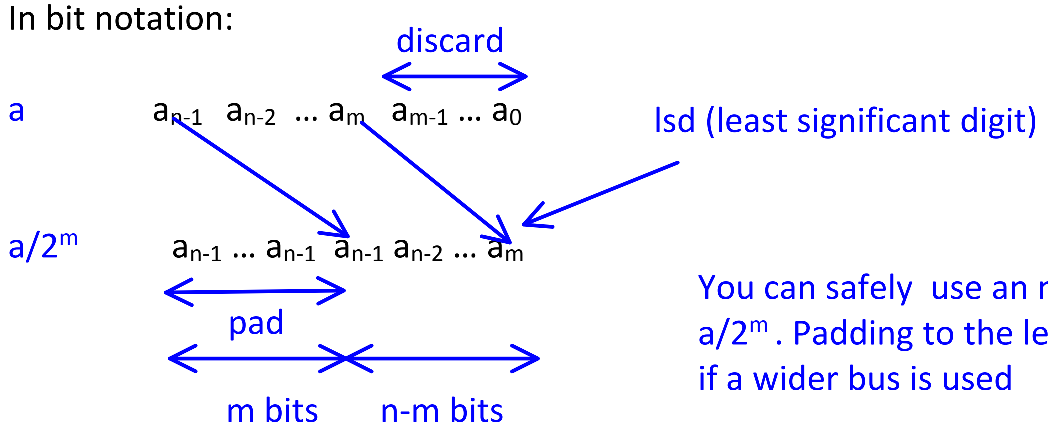
Quartus example, unsigned integer multiplication: n = 16 m = 3



Division: Signed (2's complement) Integers

For signed integers stored in 2's complement notation the procedure is exactly the same as that for unsigned integers except that we need to be carful about the sign bit. This restricts m to m <= n-2. Also we want to make sure that after shifting right by m bits the padding on the left hand side is by 0's if the sign of a is positive and by 1's if the sign is negative. The trick is to pad with the highest significant digit of a, i.e., a_{n-1} which is 0 for positive integers and 1 for negative integers.

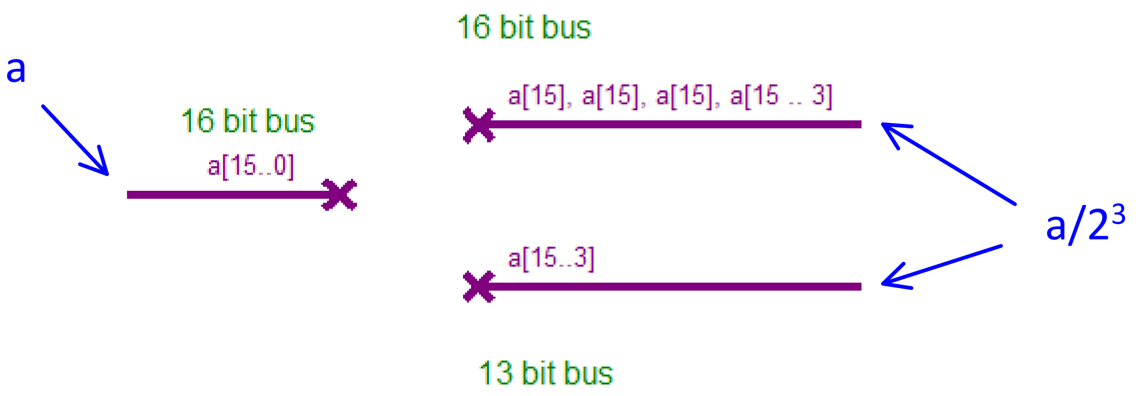
In bit notation:



You can safely use an n-m bit bus for $a/2^m$. Padding to the left is needed only if a wider bus is used

Division and multiplication by 2^m Page 4

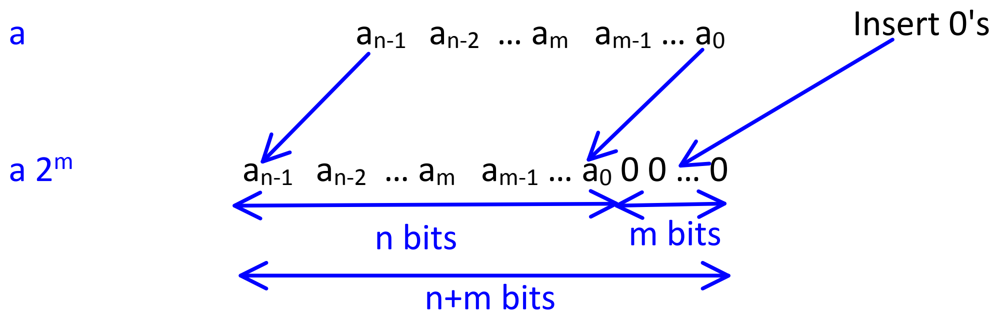
Quartus example, signed integer division: $n = 16 \quad m = 3$



Multiplication: Signed (2's complement) Integers

Just as in the case of signed division we need to make sure that if we require padding on the left then we pad with a_{n-1}, otherwise signed multiplication by 2^m is similar to unsigned multiplication.

In bit notation:



You can safely use an n+m bit bus for $a \times 2^m$. Padding to the left by a_{n-1} is needed if a wider bus is used

Division and multiplication by 2^m Page 5

Quartus example, signed integer multiplication: $n = 16$ $m = 3$