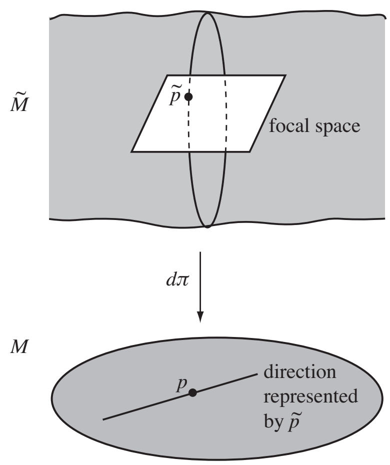
The basic construction

Suppose that M is a smooth manifold or nonsingular algebraic variety over an algebraically closed field of characteristic 0. Suppose that $\mathcal{B}$ is a rank b subbundle of its tangent bundle TM. Let $\widetilde{M} = \mathbf{P}\mathcal{B}$, the total space of the projectivization of the bundle, and let $\pi : \widetilde{M} \to M$ be the projection. A point $\widetilde{p}$ of $\widetilde{M} = \mathbf{P}\mathcal{B}$ over $p \in M$ represents a line inside the fiber of $\mathcal{B}$ at p, and since $\mathcal{B}$ is a subbundle of TM, this is a *tangent direction* to M at p. Let

$$d\pi : T\widetilde{M} \to \pi^* TM$$

denote the derivative map of π.

A tangent vector to $\widetilde{M}$ at $\widetilde{p}$ is said to be a *focal vector* if it is mapped by $d\pi$ to a tangent vector at p in the direction represented by $\widetilde{p}$; in particular a vector mapping to the zero vector (called a *vertical vector*) is considered to be a focal vector. The subspace of focal vectors is called the *focal space*.



The set of all focal vectors forms a subbundle $\widetilde{\mathcal{B}}$ of $T\widetilde{M}$, called the *focal bundle*; its rank is again b. Thus we can iterate this construction to obtain a tower of spaces (i.e., smooth manifolds or nonsingular algebraic varieties) together with their associated bundles.

If we begin the construction by taking $\mathcal{B}$ to be the tangent bundle TM itself, then the resulting tower

$$\cdots \to M(k) \xrightarrow{\pi_k} M(k-1) \xrightarrow{\pi_{k-1}} \cdots \to M(2) \to M(1) \to M(0) = M$$

is called the *Semple tower* or *monster tower* over the base M. Observe that each $M(k)$ is the total space of a $\mathbf{P}^{m-1}$-bundle over $M(k-1)$; in particular, $M(1)$ is the total space of the tangent bundle $\mathbf{P}TM$. We call $M(k)$ *monster space at level* k or the the k*th monster*. The bundle constructed at step k of the construction is called the kth *focal bundle* and denoted Δ_k; it is a subbundle of the tangent bundle $TM(k)$.