A REVIEW OF UHE NEUTRINO DETECTION USING THE ASKARYAN EFFECT

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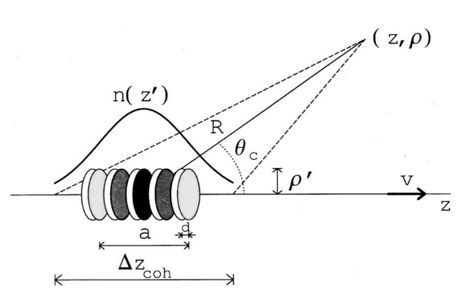
I. The Continuing Story of GZK neutrinos and Radio
II. *Regathering* our knowledge of the Askaryan effect
   A. ZHS (1991), RB (2001), ARVZ (2010-11)
   B. The basic effect, some definitions and the LPM Effect
III. Towards a complete **analytic formalism**
   A. Our implementation of RB, agreement with ZHS
   B. Accounting for the LPM effect
IV. Numerical Work
   A. Constructing the shower, given numerical constraints
   B. Deriving the general form factor provided by Geant4
V. Results in the Fourier domain, LPM and form factor together
VI. Newest work: **analytic equations in the time-domain**
   A. Graphs of asymmetric vector potential

REGATHERING OF KNOWLEDGE

The basic effect is coherent radiation from a negative charge excess:



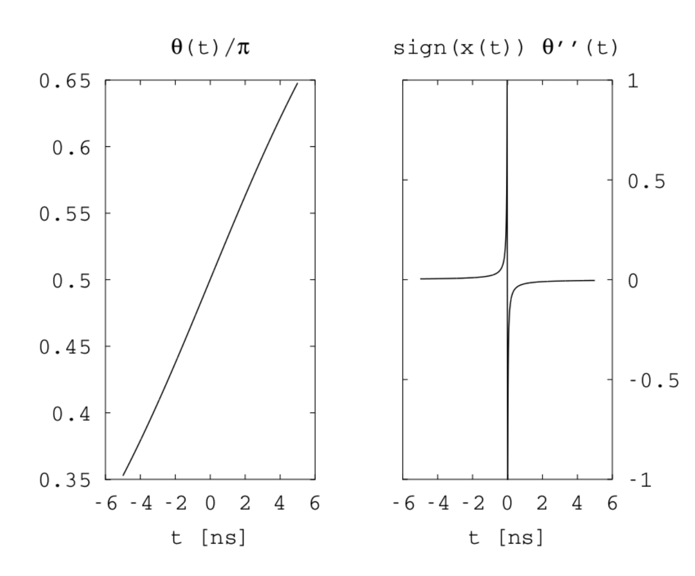
$$\eta = (a/\Delta z_{coh})^2 \qquad (1)$$

Fraunhofer regime: $E(\omega)$ has spherical symmetry ($\propto 1/R$)

Fresnel regime: $E(\omega)$ cylindrical symmetry ($\propto 1/\sqrt{\rho}$)

Feynman's formula: $E_{rad} \propto \mathrm{sgn}(1 - n\hat{u} \cdot \vec{\beta})\ddot{\theta}$

FEYNMAN'S FORMULA - IMAGINE CHARGE PASSING RIGHT TO LEFT.



Some definitions and remarks.

Longitudinal: refers to the z' coordinate, or shower axis.

Lateral: refers to the ρ' coordinate ($z^2 + \rho^2 = R^2$, $\rho/R = \sin\theta$).

θ, the viewing angle

k, wavevector in the medium

Shower width: **a (m)** $\propto \sqrt{3/2ln(E)}$

Excess charged particles: n_{max}: $\propto E/\sqrt{ln(E)}$

Energy-scaling: Product of $n_{max}a \propto E$ (area under Gaussian to first order)

Papers and other references.

ZHS - Zas, Halzen, Stanev (1991). Calculates radiation from tracks of all particles and sums them. Produces coherence argument naturally. Expensive, but proves the concept.

RB - Ralston and Buniy (2001). Completely analytic, in Fresnel and Fraunhofer regime. Factors the E-field into form factor and charge evolution.

ARZ - Alvarez-Muniz, Romero-Wolf, Zas (2010-11). Semi-analytic approach that requires simulation of charge evolution, but provides analytic formula for E-field.

Greisen parameterization (Prog. in Cosmic Ray Physics, 1956, ch. 1) E&M shower model. Leads to Rossi B appoximation etc. Solution for lateral charge evolution (see below).

$$\vec{J} = \vec{v}n(z')f(z' - ct', \vec{\rho}') \tag{2}$$

The main result from RB:

$$R\vec{E}(\omega) = 2.52 \times 10^{-7} \frac{a}{m} \frac{n_{max}}{1000} \frac{\nu}{GHz} F(\vec{q})\psi\vec{\mathcal{E}}(\theta, \eta) \tag{3}$$

$$\psi = -i \exp(ikR) \sin\theta \tag{4}$$

$$\vec{\mathcal{E}}(\theta = \theta_C, \eta) = \vec{e_\theta}(1 - i\eta)^{-1/2} \tag{5}$$

$$\vec{q} = (\omega/c, k\vec{\rho}/R) \tag{6}$$

Rossi showed that the Greissen solution for $n(z')$ with depth a **can be approximated as a gaussian with width** a

The linear ω dependence comes from acceleration factor in Lienard-Wiechert fields.

Coherence zones, and other useful approximations. The Fraunhofer approximation leads to an insight:

$$R = |\mathbf{x} - \mathbf{x}'| \gg \rho \tag{7}$$

$$R = |\mathbf{x} - \mathbf{x}'| \gg \lambda \tag{8}$$

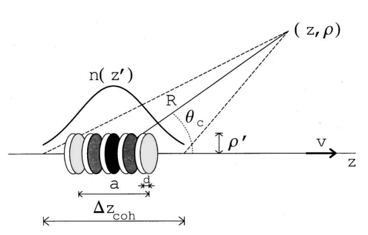
$$i|\mathbf{k}||\mathbf{x} - \mathbf{x}'| \approx \mathbf{ikR} - \mathbf{ik} \cdot \rho(\tau) \tag{9}$$

Beginning with the Lienard-Wiechert retarded potentials for decelerating charge, and focusing only on the radiation term, one can show ($y = \pi\nu\delta t(1 - n\beta\cos\theta)$):

$$\mathbf{E}(\omega, \mathbf{x}) \propto i\omega \frac{e^{ikR}}{R} \frac{\sin y}{y} \tag{10}$$

Similar to single-slit diffraction with length $\approx a$.

A more subtle approximation, keeping another order...



$$|\mathbf{x}-\mathbf{x}'| = \sqrt{(z-z')^2+(\rho-\rho')^2} \quad (11)$$

$$|\mathbf{x}-\mathbf{x}'| \approx R(z') - \frac{\rho\cdot\rho'}{R} + \left(\frac{\rho'^2}{R}\right) \quad (12)$$

$$R(z') = \sqrt{(z-z')^2+\rho^2} \quad (13)$$

Scale of the instantaneous charge excess is small compared to the longitudinal shower development. Keep the first two terms, drop the third. Integrals decouple into the **form factor,** and the **Fresnel-Fraunhofer** integrals.

The 3D Fourier transform of the charge distribution, f, the normalized charge excess distribution. (Dropping bold font for vectors).

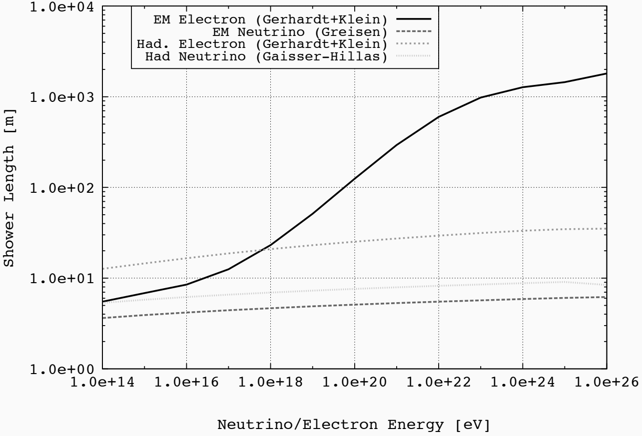
$$\int d^3x' f(x') = 1 \tag{14}$$

$$F(q) = \int d^3x' \exp\left(-iq \cdot x'\right) f(x') \tag{15}$$

$$q = \left(\frac{\omega}{c}, \frac{k}{R}\rho'\right) \tag{16}$$

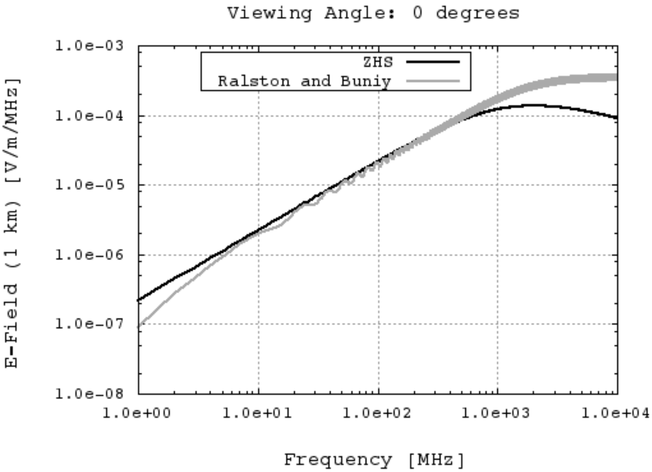
The structure of the Askaryan electric field is derived in RB, parameterized in ZHS, and fit in the time domain by ARVZ. In addition to the **LPM** effect, **the main thrust of this work is to analytically derive** $F(q)$**, and match to Monte Carlo simulations from Geant4**.

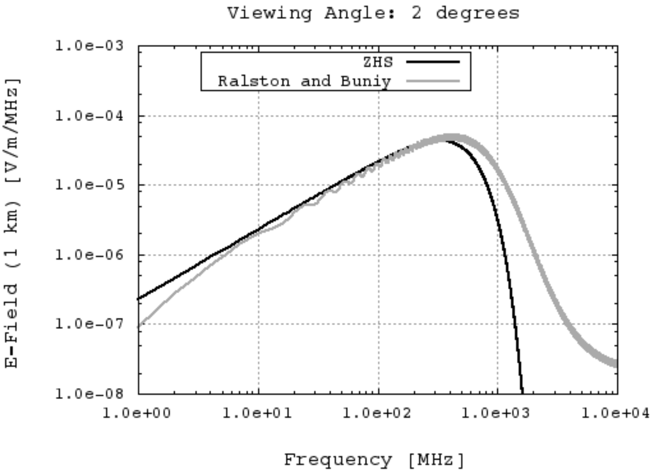
Simple incorporation: draw the a-parameter from the EM curve below, rather than Greisen.

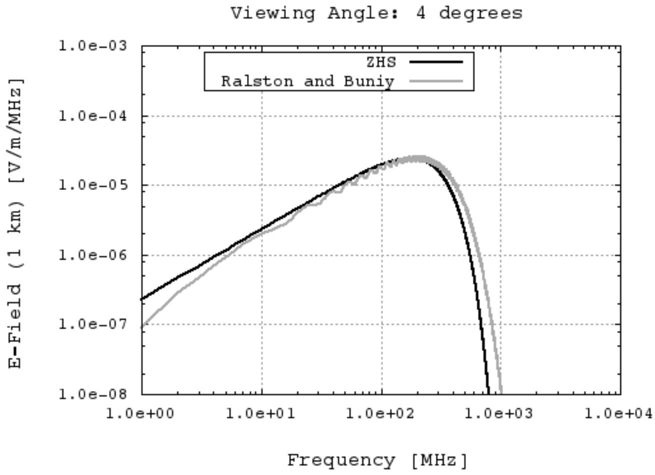


ASKARYAN FIELDS

ANALYTIC FORMS OF ASKARYAN FIELDS

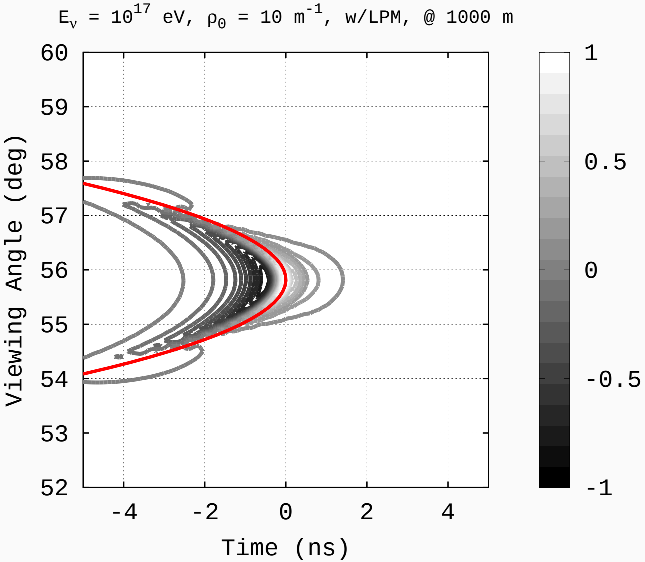



Viewing Angle: 2 degrees
ZHS
Ralston and Buniy
E-Field (1 km) [V/m/MHz]
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e+00
1.0e+01
1.0e+02
1.0e+03
1.0e+04
Frequency [MHz]

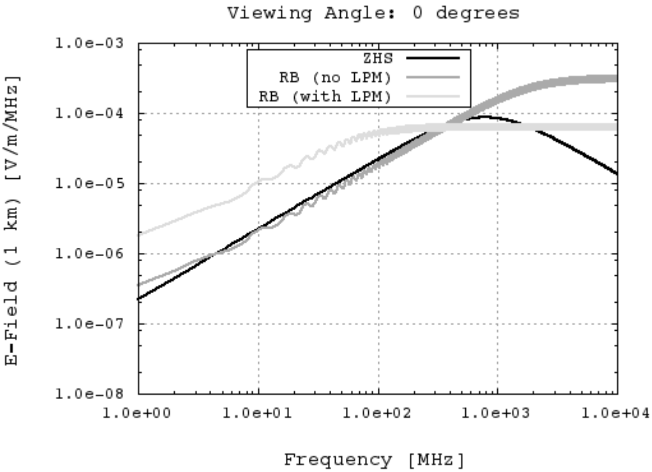


Viewing Angle: 4 degrees
ZHS
Ralston and Buniy
E-Field (1 km) [V/m/MHz]
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e+00
1.0e+01
1.0e+02
1.0e+03
1.0e+04
Frequency [MHz]

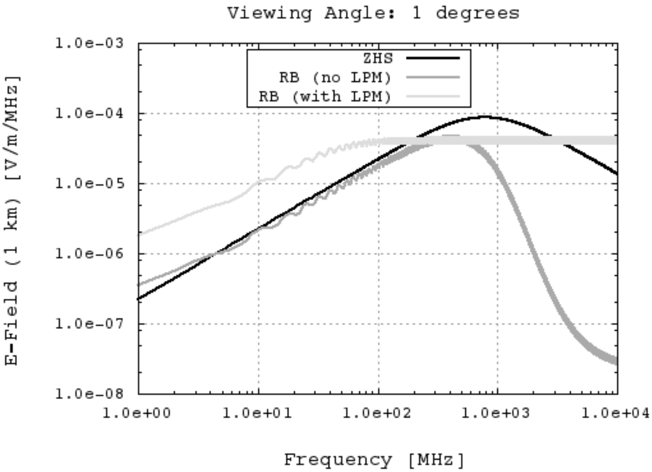
ANALYTIC FORMS OF ASKARYAN FIELDS



ACCOUNTING FOR THE LPM EFFECT - SCALING



ACCOUNTING FOR THE LPM EFFECT - SCALING



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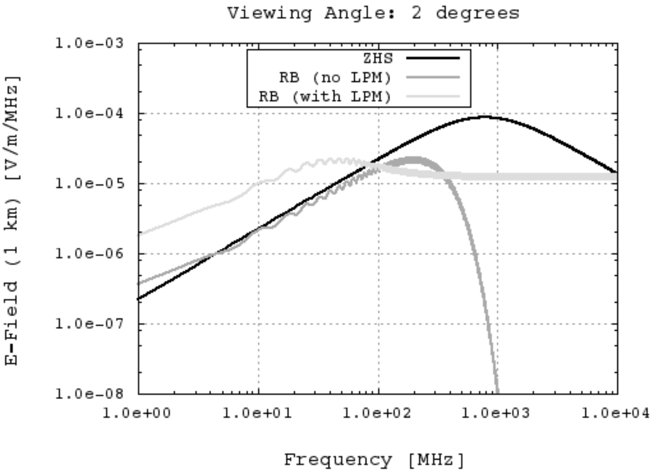
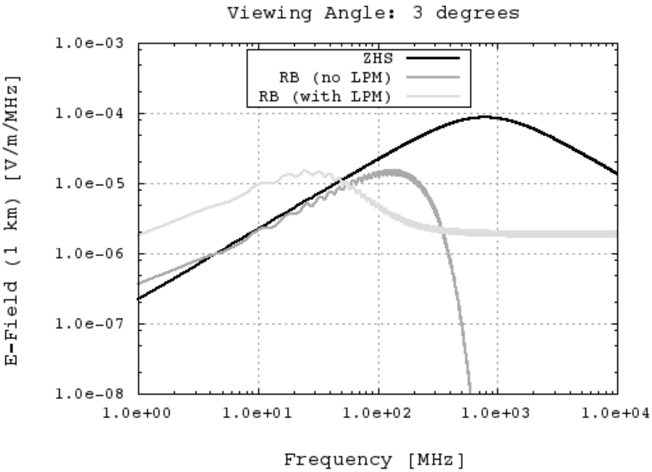
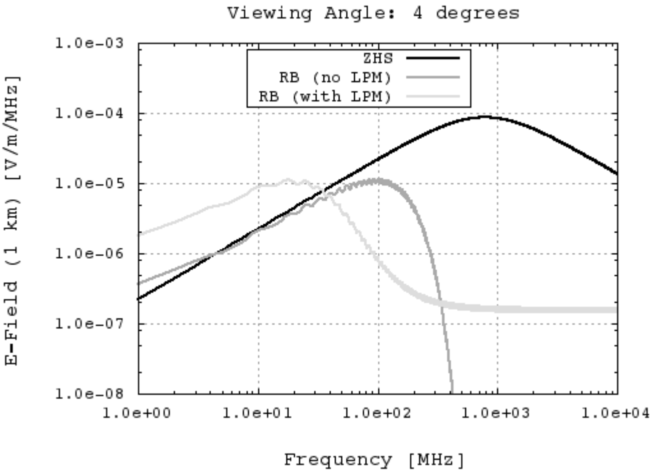
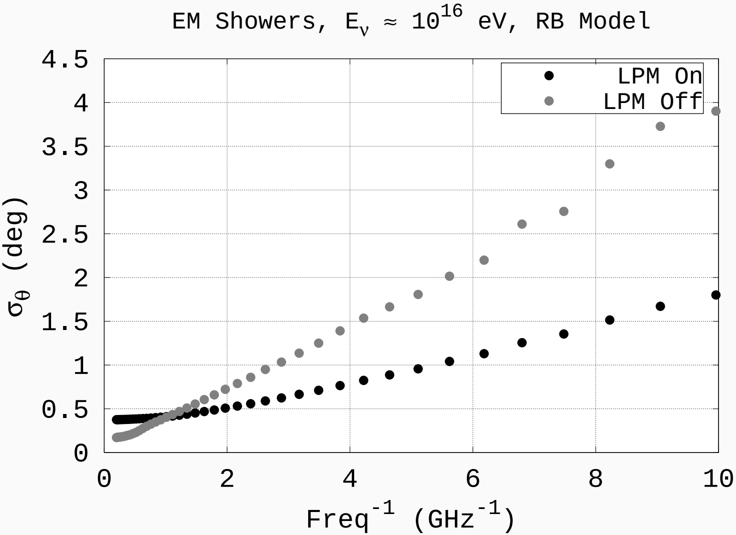


ACCOUNTING FOR THE LPM EFFECT - SCALING

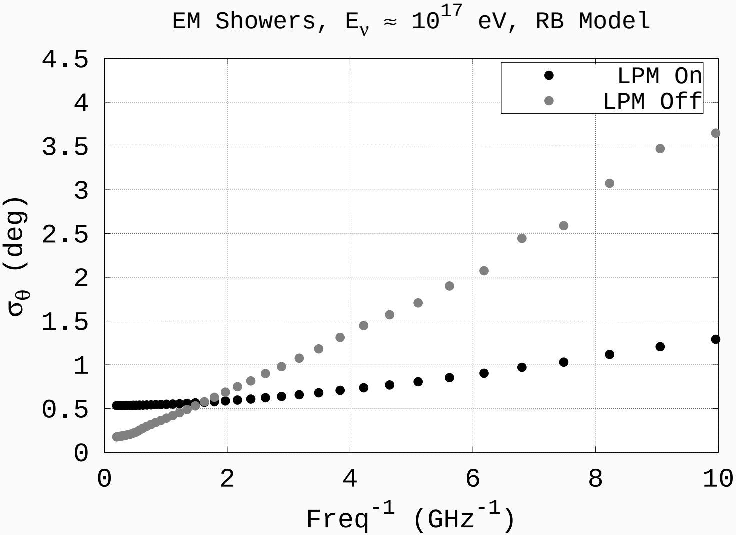



Viewing Angle: 4 degrees
ZHS
RB (no LPM)
RB (with LPM)
E-Field (1 km) [V/m/MHz]
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e+00
1.0e+01
1.0e+02
1.0e+03
1.0e+04
Frequency [MHz]

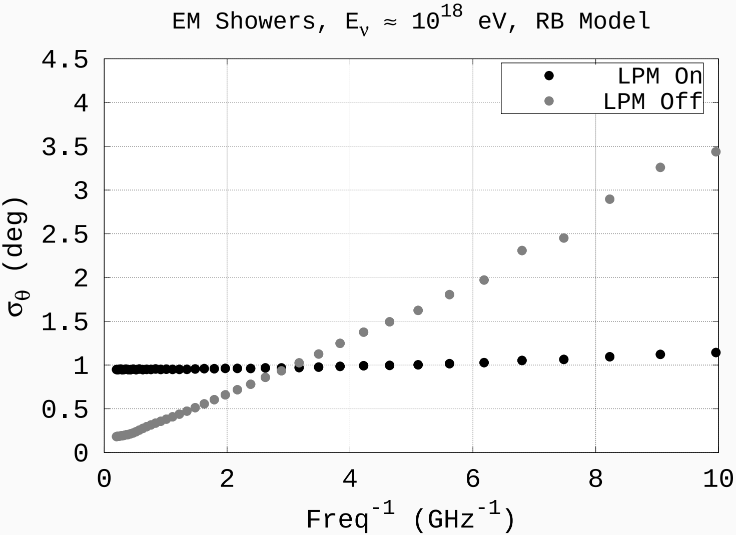
ACCOUNTING FOR THE LPM EFFECT - VIEWING ANGLE



ACCOUNTING FOR THE LPM EFFECT - VIEWING ANGLE



ACCOUNTING FOR THE LPM EFFECT - VIEWING ANGLE



NUMERICAL WORK

Limitations:

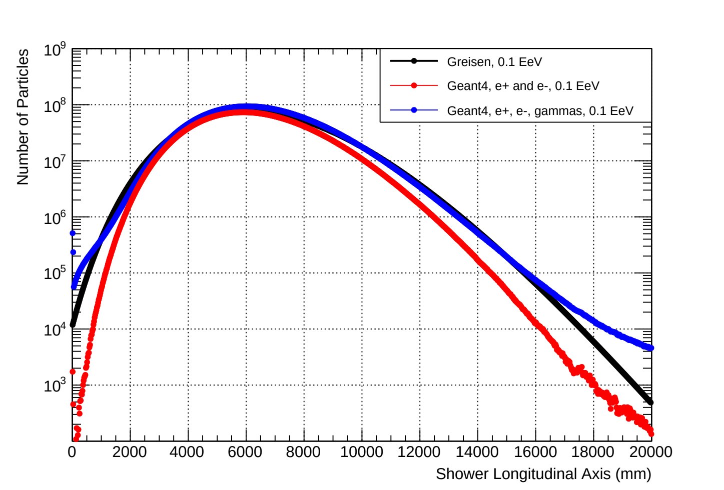
- Few hundred jobs at once.
- Charged RUs from finite account, 1 RU = 10 CPU-hours
- Memory use < 8 GB, (MC thresholds)

Strategy: Implement pre-shower sub-shower strategy, with Geant4.

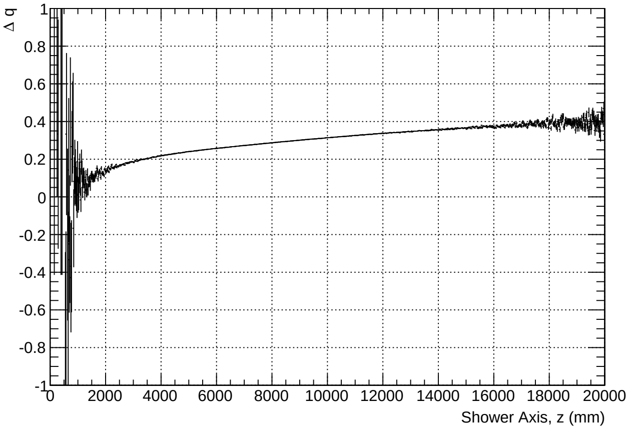
- Utilizes **back-fill** (each sub-shower is 10 cpu-minutes).
- Obeys memory constraints
- Cost: few hundred RUs, courtesy of Dr. Amy Connolly, @ OSU

Goal: $F(\omega, \theta)$ **using Geant4 pre-showers and sub-showers**.

GEANT4 SIMULATIONS

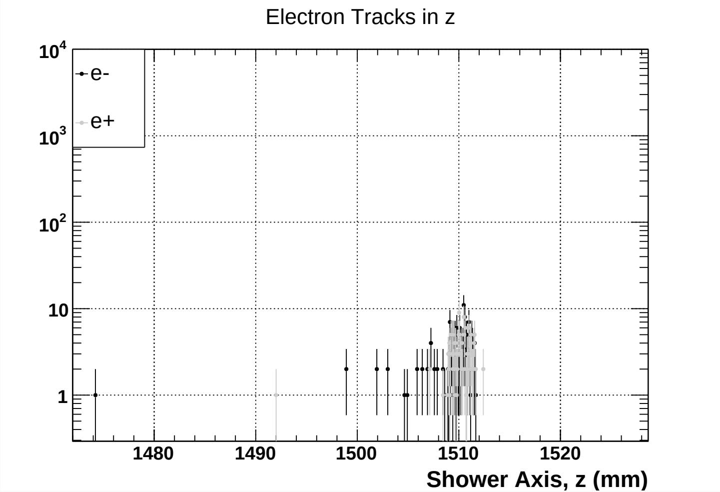


Fractional negative charge excess, Δq. (MC threshold-dependent)

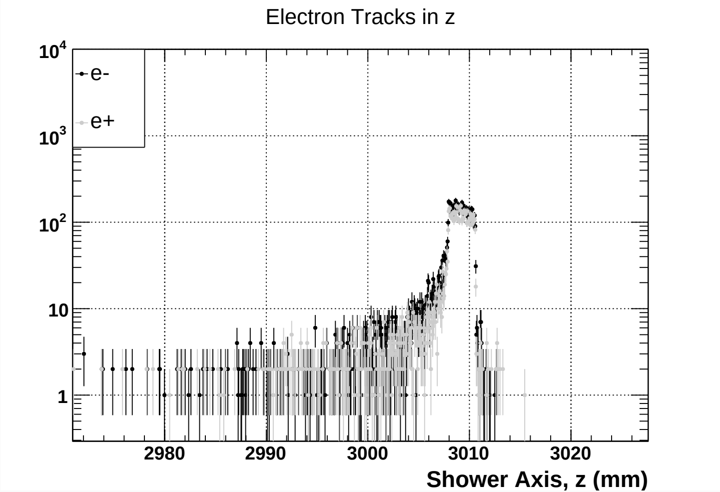


GEANT4 SIMULATIONS - Z'-FORM FACTOR DEPENDENCE?

All tracks in 10 ps window @ 5 ns after primary interaction:

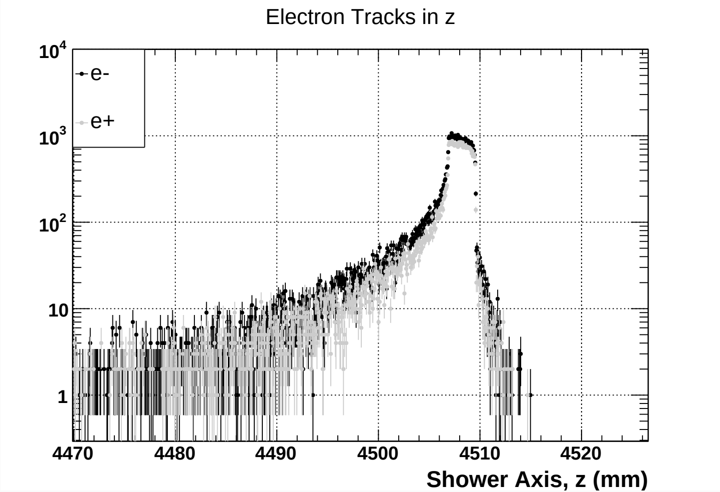


All tracks in 10 ps window @ 10 ns after primary interaction:

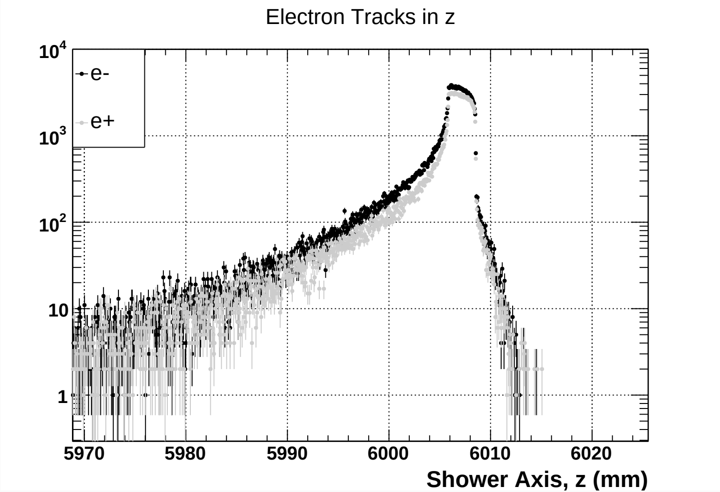


GEANT4 SIMULATIONS - Z'-FORM FACTOR DEPENDENCE?

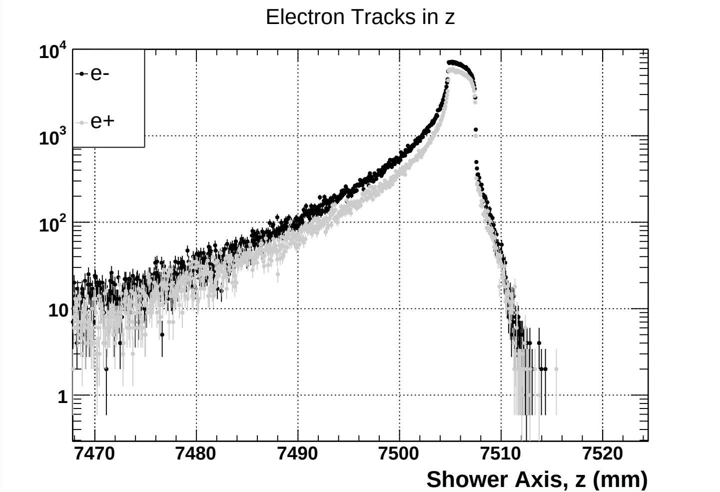
All tracks in 10 ps window @ 15 ns after primary interaction:



All tracks in 10 ps window @ 20 ns after primary interaction:

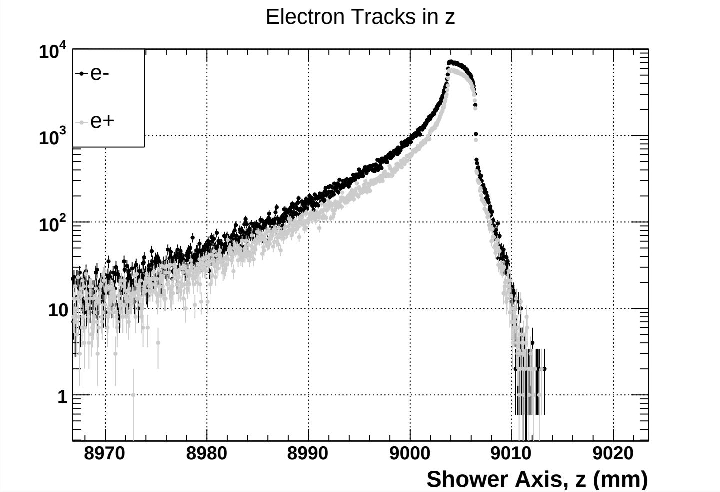


All tracks in 10 ps window @ 25 ns after primary interaction:



GEANT4 SIMULATIONS - Z'-FORM FACTOR DEPENDENCE?

All tracks in 10 ps window @ 30 ns after primary interaction:



The "instantaneous" form factor in z' is so small, it doesn't limit the Askaryan radiation...

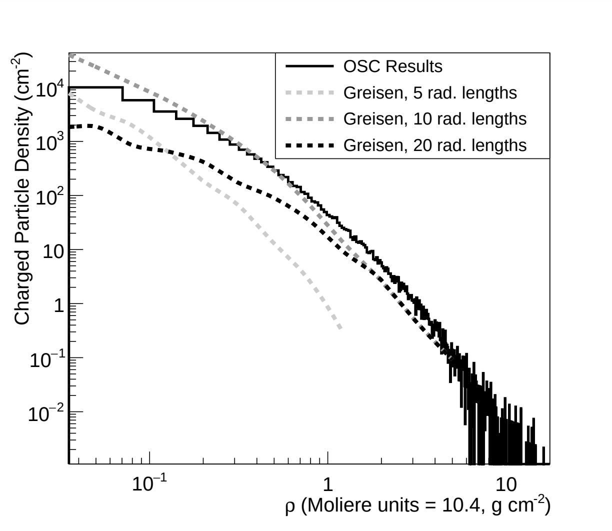
Unless the time-scale that matters is actually the Nyquist frequency of the RF detectors (1 GHz or 1 ns).

If that were true, then the z-shape could matter (long tail, flat top is limited by time-window).

One can show that the phase shift due to any z-dependence in form factors goes like

$$\phi/\Delta\theta \approx 2\pi n \left\{ \frac{\nu \Delta t}{\Delta\theta} - \frac{R}{\lambda} \sin\theta_C \right\} \tag{17}$$

GEANT4 SIMULATIONS - ρ'-FORM FACTOR DEPENDENCE



Necessary to explain why deccelerating charge doesn't radiate up to optical frequencies: $E(k) \approx k$.

$$\boxed{F_{ZHS}(k) = \frac{1}{1 + \left(\frac{k}{k_0}\right)^2} = \frac{k_0^2}{k_0^2 + k^2}} \tag{18}$$

What does the corresponding charge distribution (inverse Fourier transform) resemble? Must treat the poles carefully.

$$f(\rho') = \frac{k_0^2}{2\pi} \int_{-\infty}^{\infty} \frac{e^{ik\rho'}}{(k + ik_0)(k - ik_0)} dk, k \in \mathbb{C} \tag{19}$$

$$f(\rho')/k_0^2 = \frac{1}{2\pi i} \oint \frac{ie^{ik\rho'}(k + ik_0)^{-1}}{(k - ik_0)} \tag{20}$$

$$f(\rho')/k_0^2 = \frac{1}{2\pi i}\oint \frac{ie^{ik\rho'}(k+ik_0)^{-1}}{(k-ik_0)} \tag{21}$$

$$f(\rho') = k_0^2\left(ie^{ik\rho'}(k+ik_0)^{-1}\right)_{k=ik_0} \tag{22}$$

$$f(\rho') = \frac{k_0}{2}e^{-k_0\rho'} \tag{23}$$

Exponential (interesting), normalized to $\frac{1}{2}$. Using the oppositely oriented contour, we get a different distribution:

$$f(\rho') = -k_0^2\left(ie^{ik\rho'}(k-ik_0)^{-1}\right)_{k=-ik_0} = \frac{k_0}{2}e^{k_0\rho'} \tag{24}$$

We must choose the contour that follows Jordan’s lemma - (see below).

What about a form factor like:

$$f(\rho') = k_0 \exp(-k_0 z'), \rho' > 0 \tag{25}$$

Fourier transform gives the form factor:

$$\boxed{F_{JCH}(k) = \int_{-\infty}^{\infty} dz' e^{-ik\rho'} k_0 e^{-k_0\rho'} = \frac{k_0}{k_0 - ik}} \tag{26}$$

Only one pole, at $k = -ik_0$, and still cuts off the high-frequency spectrum. *So in the 1D case, just remove a pole, or, take the contour that converges.*

Taking the inverse Fourier transform of F_{JCH} requires closing the contour around the one pole, and using Cauchy's formula.

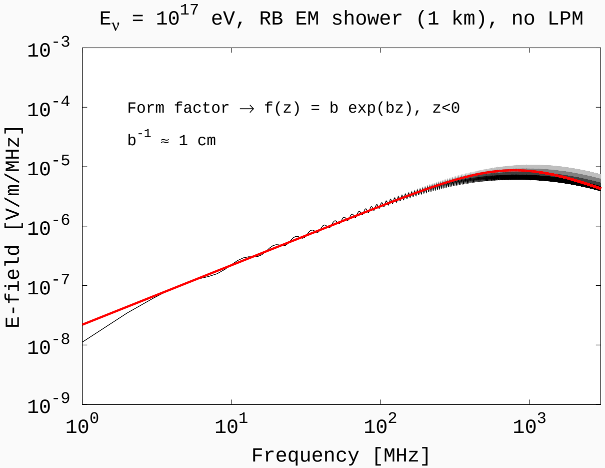
$$f(\rho')/k_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{ik\rho'}}{k_0 - ik} dk \tag{27}$$

$$f(\rho')/k_0 = \frac{1}{2\pi i}\oint \frac{ie^{ik\rho'}}{k_0 - ik} dk = \frac{1}{2\pi i}\oint \frac{e^{ik\rho'}}{k - ik_0} dk \tag{28}$$

$$f(\rho') = k_0 e^{-k_0\rho'},\ \rho' > 0 \tag{29}$$

Notice that $\Re(F_{JCH}) = F_{ZHS}$, and $|F_{JCH}| = \sqrt{F_{ZHS}}$ (for same k_0). This means that my form factor also cuts off the spectrum at high frequencies, and reduces to ZHS if we ignore imaginary E before taking the magnitude. Interesting that $\arg(F_{JCH}) \approx k/k_0$, so k_0 should be large, to avoid adding extraneous phases.

RESULT



$E_\nu = 10^{17}$ eV, RB EM shower (1 km), no LPM
Form factor → f(z) = b exp(bz), z<0
$b^{-1} \approx 1$ cm
E-field [V/m/MHz]
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency [MHz]

For the general, 3D case, I propose a ρ'-dependence as follows:

$$f(x') = f_0\delta(z')\exp(-\sqrt{2\pi}\rho_0\rho'), \quad \int dz' d^2\rho' f(x') = 1, \;\; f_0 = \rho_0^2 \tag{30}$$

$$F(q) = \int_{-\pi}^{\pi}\int_{0}^{\infty}\int_{\infty}^{\infty} dz'\rho' d\rho' d\phi' e^{-iq\cdot x'} f(x') \tag{31}$$

$$\gamma = k\sin\theta \;\; (m^{-1}) \tag{32}$$

$$\sigma = \frac{\gamma}{\sqrt{2\pi}\rho_0} \tag{33}$$

So σ is the ratio of the lateral projection of the wave-vector and the lateral charge extent. Perform z'-integration and substitute:

$$F(q) = \rho_0^2 \int_0^{\infty} \rho' d\rho' \int_{-\pi}^{\pi} d\phi' \exp\{-(i\gamma\cos\phi + \gamma/\sigma)\rho'\} \tag{34}$$

Shift $\phi \to \phi - \pi/2$ (cylindrical symmetry), and perform ϕ-integration:

$$F(q) = \rho_0^2 \int_0^\infty \rho' d\rho' \int_{-\pi}^{\pi} d\phi' \exp\{-(i\gamma \cos\phi + \gamma/\sigma)\rho'\} \tag{35}$$

$$F(q) = \rho_0^2 \int_0^\infty \rho' d\rho' \exp\left\{-\frac{\gamma}{\sigma}\rho'\right\} \int_{-\pi}^{\pi} d\phi' \exp\{-i\gamma\rho' \sin\phi\} \tag{36}$$

$$F(q) = 2\pi\rho_0^2 \int_0^\infty d\rho' \rho' \exp\left\{-\frac{\gamma}{\sigma}\rho'\right\} J_0(\gamma\rho') \tag{37}$$

$$F(q) = \sigma^{-2} \int_0^\infty du' u' \exp\{-u'/\sigma\} J_0(u') \tag{38}$$

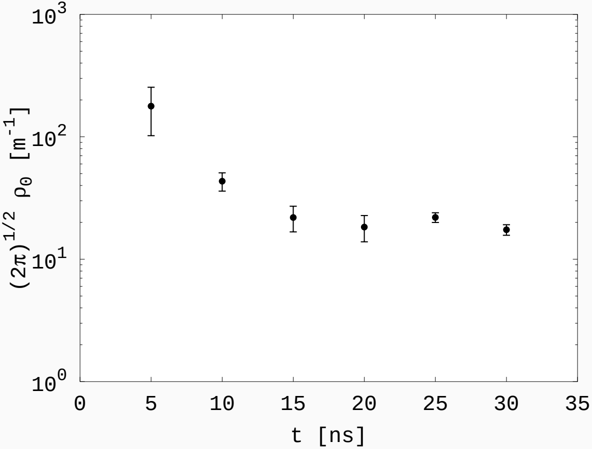
Table of integrals...and finally:

$$\boxed{F(k,\theta) = \frac{1}{(1+\sigma^2)^{3/2}} = \left(1 + \left(\frac{k}{\rho_0}\right)^2 \left(\frac{\sin\theta}{2\pi}\right)^2\right)^{-3/2}} \tag{39}$$

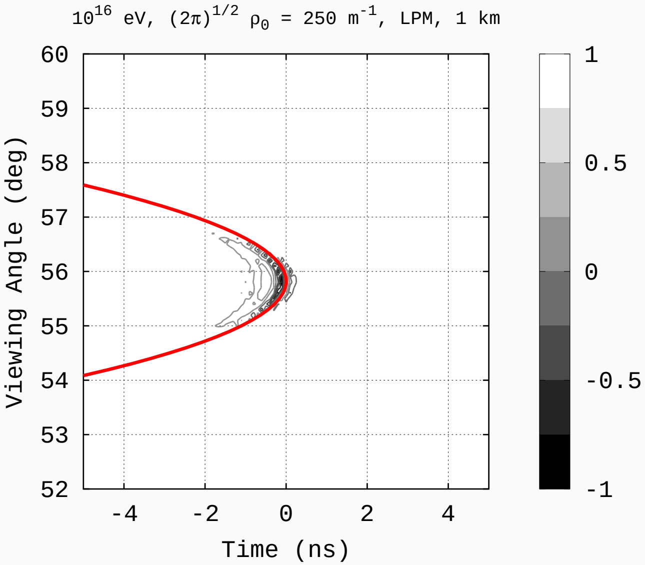
COMBINED RESULTS

ρ'-form factor dependence - Monte Carlo fit parameters

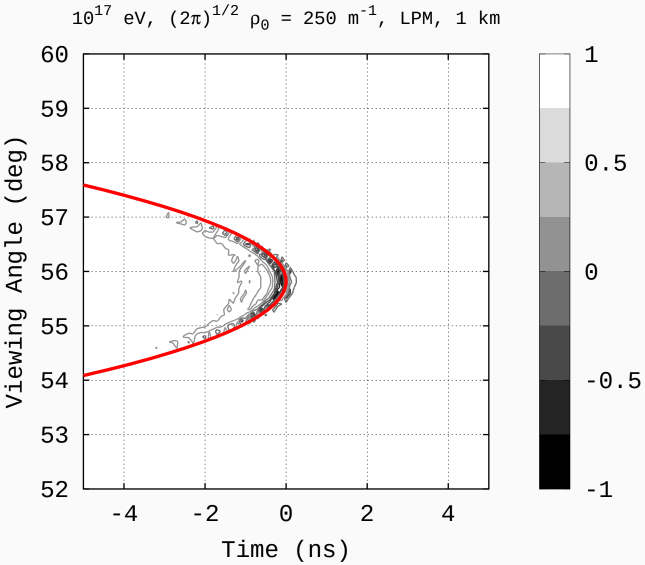
Shower energy: 10^{17} eV.



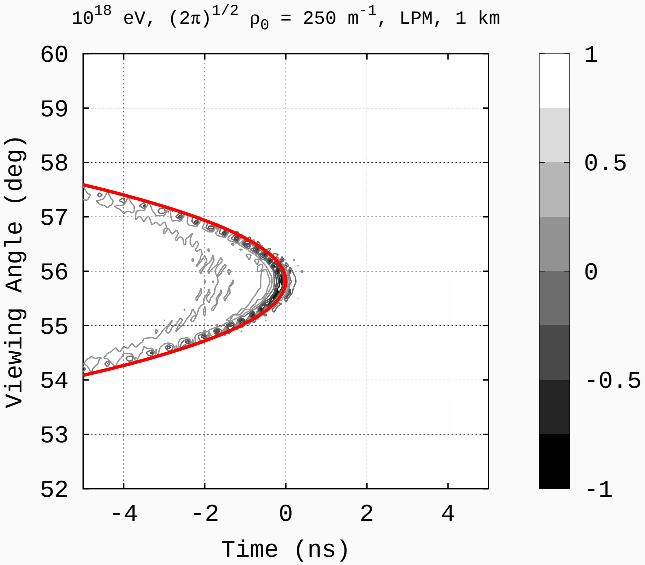
COMPLETE FIELD - θ POLARIZATION

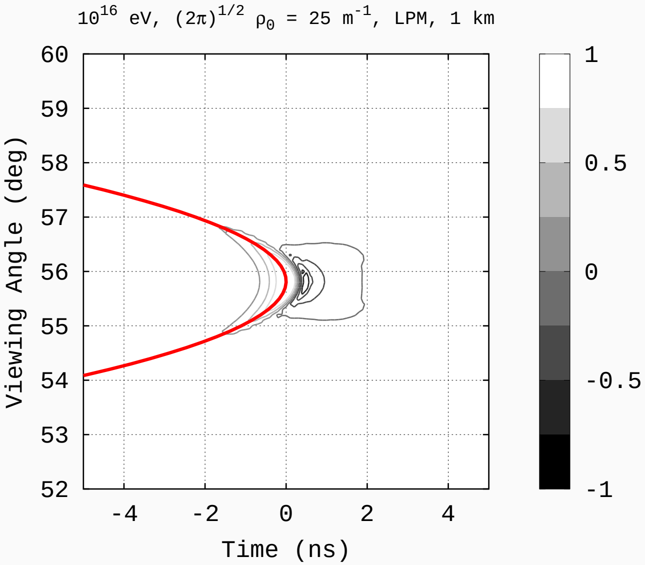


COMPLETE FIELD - θ POLARIZATION



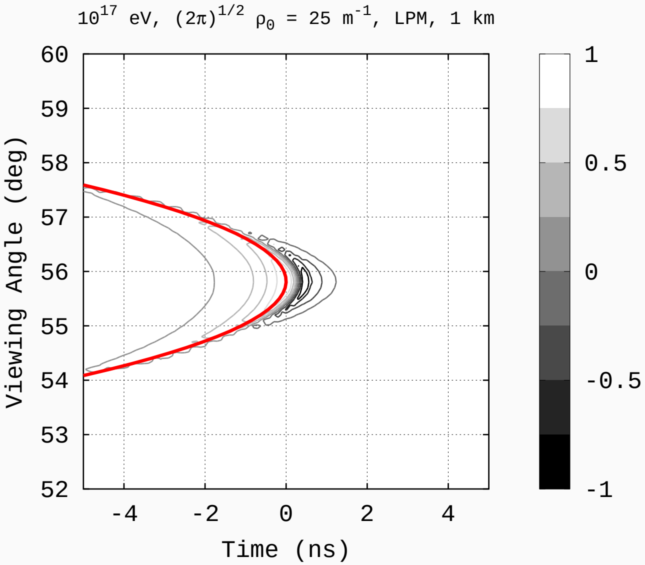
COMPLETE FIELD - θ POLARIZATION



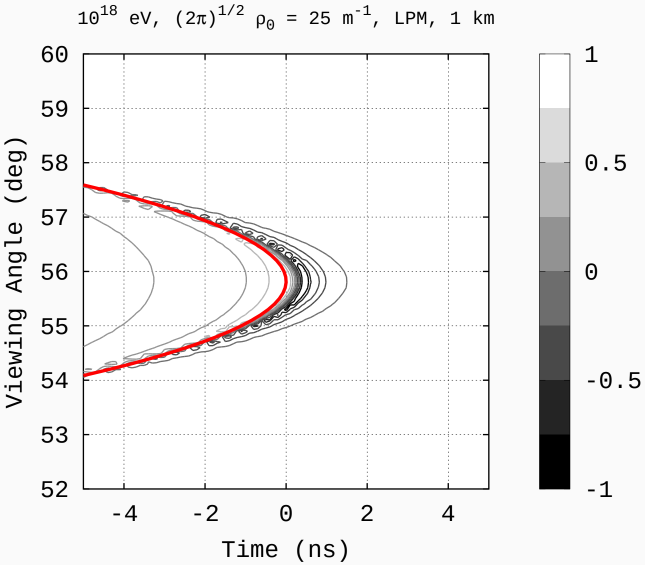


10^{16} eV, $(2\pi)^{1/2}$ ρ_0 = 25 m^{-1}, LPM, 1 km
Viewing Angle (deg)
60
59
58
57
56
55
54
53
52
-4
-2
0
2
4
Time (ns)
1
0.5
0
-0.5
-1

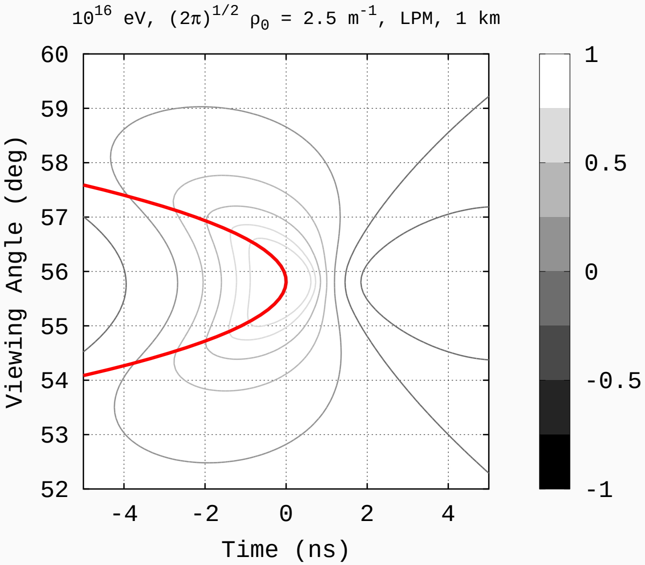
COMPLETE FIELD - θ POLARIZATION



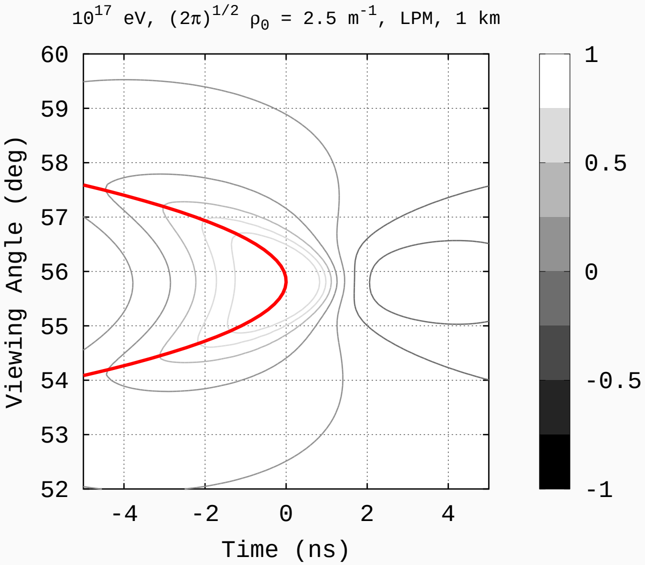
COMPLETE FIELD - θ POLARIZATION



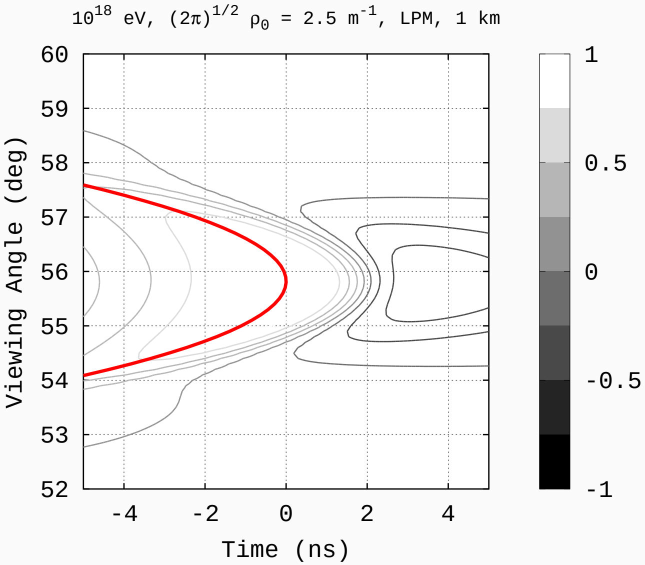
COMPLETE FIELD - θ POLARIZATION



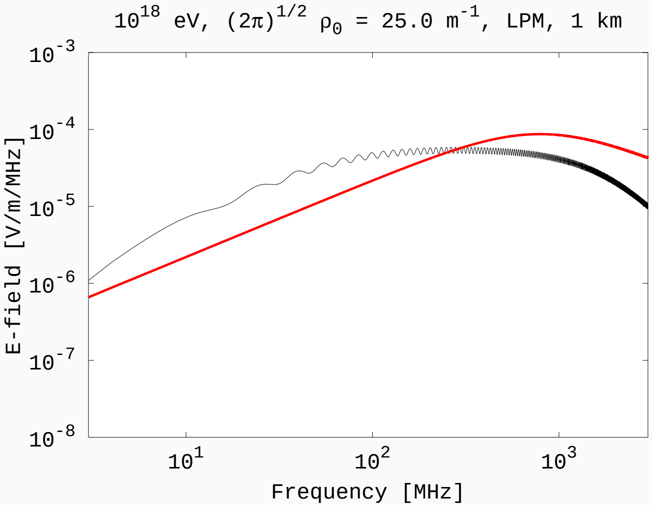
COMPLETE FIELD - θ POLARIZATION



COMPLETE FIELD - θ POLARIZATION



SPECTRA - θ POLARIZATION



NEW RESULTS - PURELY ANALYTIC TIME-DOMAIN FIELDS

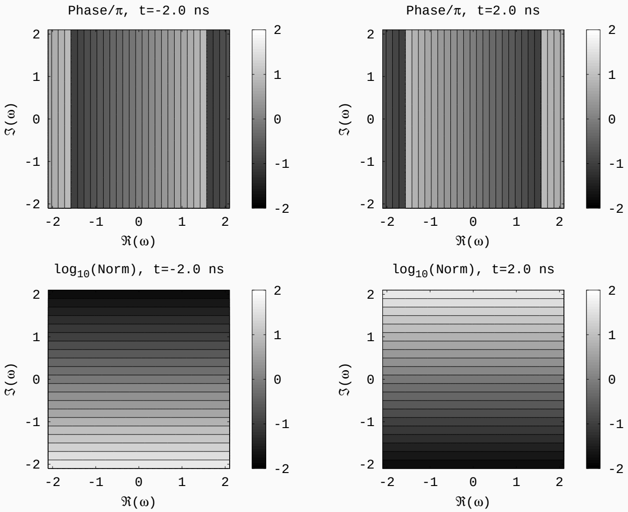
At the Cerenkov cone, with ideal form factor ($F(\omega, \theta) = 1$), the E-field takes a convenient form:

$$R\mathbf{E}(\omega, \theta_C) = -\frac{i\omega E_0 \sin\theta_C e^{i\omega R/c}}{(1 - i\eta)^{1/2}} \hat{e}_\theta \quad [\mathrm{V/Hz}] \tag{40}$$

- E_0: Energy scaling-normalization (goes as $n_{max}a$, or the area under the Gaussian $n(z')$, i.e. total charged particles).
- $\eta = k(a^2 \sin^2\theta)/R$, same parameter from RB, $\eta = \omega/\omega_C$.
- $\hat{e}_\theta$ is the spherical unit vector (field is linearly polarized orthogonal to viewing direction).

$$R\mathbf{E}(t_r, \theta_C) \approx \frac{i\omega_C E_0 \sin\theta_C}{\pi} \hat{e_\theta} \frac{d}{dt_r} \oint_C d\omega \frac{e^{-it_r\omega}}{\omega + 2i\omega_C} \tag{41}$$

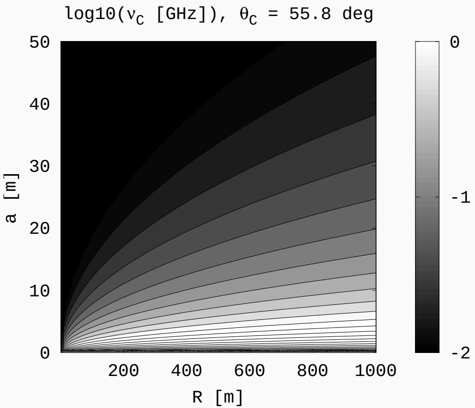
JORDAN'S LEMMA, INVERSE FOURIER TRANSFORM



$$R\mathbf{E}(t_r, \theta_C) \approx 4E_0 \sin(\theta_C)\omega_C^2 \hat{e_\theta} \begin{cases} \exp(2\omega_C t_r) \quad [\mathrm{V}] & t_r \leq 0 \\ -\exp(-2\omega_C t_r) \quad [\mathrm{V}] & t_r > 0 \end{cases} \tag{42}$$

Under the Lorentz gauge condition for Maxwell's equations, in the absence of any static potentials, the negative derivative of the vector potential yields the electric field: $-\partial\mathbf{A}/\partial t = \mathbf{E}$.

$$R\mathbf{A}(t_r, \theta_C) \approx -2E_0\omega_C \sin\theta_C \hat{e}_\theta \begin{cases} \exp(\omega_C t_r) \quad [\mathrm{V}\cdot\mathrm{s}] & t_r \leq 0 \\ \exp(-\omega_C t_r) \quad [\mathrm{V}\cdot\mathrm{s}] & t_r > 0 \end{cases} \tag{43}$$



$$\boxed{\nu_C = \frac{cR}{2\pi a^2 \sin^2\theta_C}} \tag{44}$$

Let $\sigma = \omega/\omega_{CF}$.

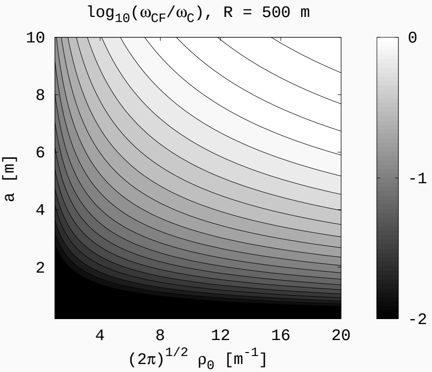
$$R\mathbf{E}(\omega, \theta_C) = -\frac{i\omega E_0 \sin\theta_C e^{i\omega R/c}}{(1 - i\omega/\omega_C)^{1/2}(1 + (\omega/\omega_{CF})^2)^{3/2}} \hat{e_\theta} \quad [\mathrm{V/Hz}] \tag{45}$$

In the limit $\sigma < 1$, and $\eta < 1$, $\omega_0 = 2/3\ \omega_{CF}$.

$$R\mathbf{E}(t_r, \theta_C) \approx \frac{2i}{3\pi} \hat{e_\theta} \frac{d}{dt_r} \oint d\omega \frac{E_0 \sin\theta_C \omega_{CF}^2 \omega_C e^{-it_r\omega}}{(\omega + 2i\omega_C)(\omega + i\omega_0)(\omega - i\omega_0)} \quad [\mathrm{V}] \tag{46}$$

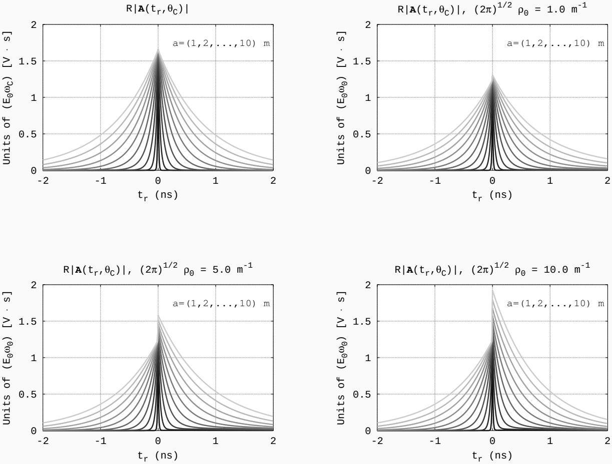
Key figure of merit: ratio of form factor limiting frequency, and the coherence limiting frequency:

$$\boxed{\epsilon' = (\sqrt{2\pi}\rho_0\rho)\left(\frac{a}{R}\right)^2} \tag{47}$$

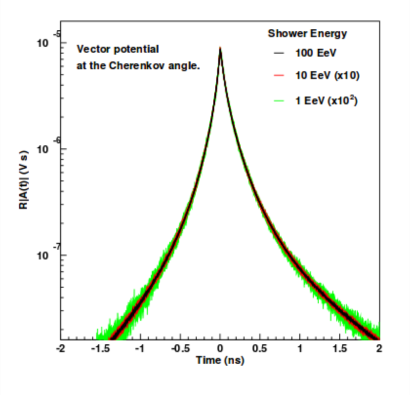


$$\boxed{\epsilon' = (\sqrt{2\pi}\rho_0\rho)\left(\frac{a}{R}\right)^2} \tag{48}$$

FINAL RESULTS (VECTOR POTENTIAL)



FOR COMPARISON TO ARVZ (2011)



Our model reproduces the shape, width, and asymmetry, of the semi-analytical approach. (Taylor expansion of ω_C exponential gets the form of ARVZ $A(t, \theta_C)$).

CONCLUSIONS

I. Developed a fully analytic RB-model of Askaryan radiation
   A. Accounts for LPM effect, and $F(\omega, \theta)$.
   B. $F(\omega, \theta)$ was derived with the help of the Greisen parameterization, Geant4, and the OSC
II. Results in the Fourier domain, LPM and form factor together
III. Newest work: **analytic equations in the time-domain**
IV. This work will be published in a forthcoming paper, and posted on arXiv by April 23

The code is on github:

$\boxed{git\ clone\ https://github.com/918particle/AraSim2\ AraSim2}$

CONCLUSIONS - DEVELOPERS NEEDED FOR ARASIM2