Review Session 12/7 Answers:

1. $f(x+h) = (x+h)^2 + 3(x+h) = x^2+2xh+h^2+3x+3h$

$$\frac{\begin{aligned} f(x+h) &= x^2+2xh+h^2+3x+3h \\ - \quad f(x) &= x^2+3x \end{aligned}}{f(x+h)-f(x) = 2xh+h^2+3h}$$

$$\Rightarrow \frac{f(x+h)-f(x)}{h} = \boxed{2x+h+3}$$

2(a). not defined when denom $=0$.

$|x+4|+2=0$ when $|x+4|=-2$.

$|\cdot| \geq 0$ ALWAYS, so $|x+4|$ may NEVER be -2,

So $|x+4|+2 \neq 0$ for all x and

domain of f is $(-\infty, \infty)$

(b). not defined when denom $=0$

$|x+4|-2=0 \Rightarrow |x+4|=2 \Rightarrow x+4=\pm 2 \Rightarrow \boxed{\begin{aligned} &x=-2 \\ \text{or } &x=-6 \end{aligned}}$

$\Rightarrow$ domain of g is $(-\infty,-6)\cup(-6,-2)\cup(-2,\infty)$

(c). $\log(\cdot)$ requires inside >0, so we need

$$\frac{(x+7)^3(7-x)}{x^3-3x} = \frac{(x+7)^3(7-x)}{x(x^2-3)} = \frac{(x+7)^3(7-x)}{x(x-\sqrt{3})(x+\sqrt{3})} > 0.$$

Use a number line / chart

$\frac{(-)(+)}{(-)(-)(-)} = +$

$+$		$-$		$+$		$-$		$+$		$-$
	-7		$-\sqrt{3}$		0		$\sqrt{3}$		7	
	zero mult 3		pole mult 1		pole mult 1		pole mult 1		zero mult 1	

$\Rightarrow$ domain is $(-\infty,-7)\cup(-\sqrt{3},0)\cup(\sqrt{3},7)$

(d). $\sec x = \frac{1}{\cos x}$; undef'd when $\cos x = 0$; i.e. odd mults of $\frac{\pi}{2}$

domain$(j) = \{x \mid x \neq (2k+1)\frac{\pi}{2},\ k \text{ any integer}\}$

(e). Treat $x+\frac{\pi}{4}$ as a variable u. We want to know when $\tan u = \frac{\sin u}{\cos u}$ is undefined — this happens when $\cos u = 0$, i.e. $u = (2k+1)\frac{\pi}{2}$, k any integer

$$x+\frac{\pi}{4} = (2k+1)\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{4} + (2k+1)\frac{\pi}{2}.$$

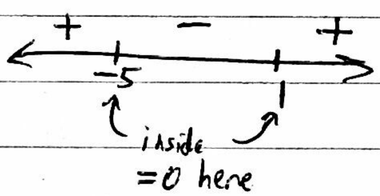
$$\text{domain}(k) = \{x \mid x \neq -\tfrac{\pi}{4} + (2k+1)\tfrac{\pi}{2},\ k \text{ any integer}\}$$

(f). $\sqrt{\ }$ requires inside ≥ 0.

So, need $x^2+4x-5 \geq 0$ →

$(x+5)(x-1) \geq 0$

$\Rightarrow \boxed{(-\infty,-5]\cup[1,\infty)}$



(g). quotient: intersect the following:

- domain$(a) = \{x \neq \pm 2\}$
- domain$(b) = \{x \neq 2\}$
- $\{b(x) \neq 0\} = \{x \neq 0\}$

$\Rightarrow \text{domain}\left(\frac{a}{b}\right) = \{x \neq \pm 2, 0\} = (-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

(h). First, compute $(b\circ a)(x)$, simplify

$$(b\circ a)(x) = \frac{\left(\frac{5}{x^2-4}\right)}{\left(\frac{5}{x^2-4}\right) - 2\cdot\frac{x^2-4}{x^2-4}} = \frac{\left(\frac{5}{x^2-4}\right)}{\left(\frac{5-2x^2+8}{x^2-4}\right)} = \frac{5}{13-2x^2} \quad \text{if } x \neq \pm 2.$$

$$13-2x^2 = 0 \Rightarrow x^2 = \frac{13}{2} \Rightarrow x = \pm\sqrt{\frac{13}{2}}$$

So, need $x \neq \pm\sqrt{\frac{13}{2}}$ & x in domain$(a) = \{x \neq \pm 2\}$

$$\boxed{\text{domain}(n) = \left(-\infty,-\sqrt{\tfrac{13}{2}}\right)\cup\left(-\sqrt{\tfrac{13}{2}},-2\right)\cup(-2,2)\cup\left(2,\sqrt{\tfrac{13}{2}}\right)\cup\left(\sqrt{\tfrac{13}{2}},\infty\right)}$$

(i) intersect domain$(c) = \{x \neq \pm 2\}$ & domain$(d) = \{x \neq 5\}$

$$\boxed{\text{domain}(p) = (-\infty,-2)\cup(-2,2)\cup(2,5)\cup(5,\infty)}$$

Cancelation is irrelevant in calculating domain!

Problem 3: $\dfrac{e^5 - 1}{5 - 0}$

Problem 4: (a). $-(2(x+3))^2$
(b). $\left[-(\tfrac{1}{2}x+3)\right]^2$

Problem 5: (a). odd ; (b). even ; (c). neither; (d) even ; (e). even ; (f). neither;
(g). odd

Problem 6: (a). $5(x+1)^2+4$; min @ vertex $(-1,4)$, range: $[4,\infty)$ no x-ints
(b). $-3(x+\tfrac{1}{2})^2+\tfrac{7}{4}$; max @ vertex $(-\tfrac{1}{2},\tfrac{7}{4})$, range: $(-\infty,\tfrac{7}{4}]$ two x-ints

Problem 7: (a). Zeros @ $\tfrac{5}{2}, 3, \tfrac{7}{6}$

$-$ $\tfrac{7}{6}$ $+$ $\tfrac{5}{2}$ $-$ 3 $+$

degree 3 w/ positive leading term $4(2x)(-x)(-6x) = 48x^3$
$\Longrightarrow$ $f(x) \to -\infty$ as $x \to -\infty$
and $f(x) \to \infty$ as $x \to \infty$.

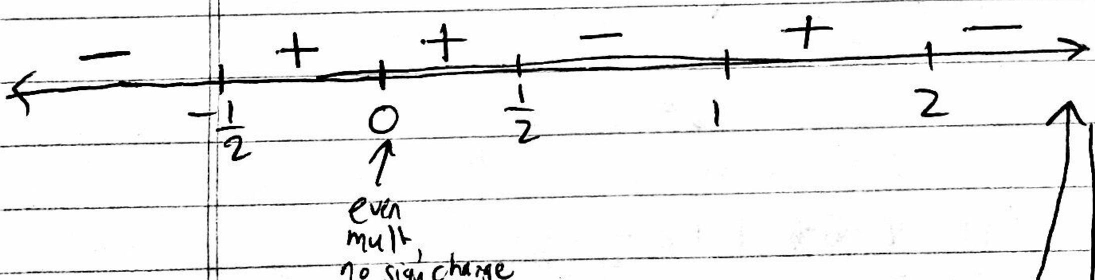
(b) degree (top) = degree (bottom)
$\Longrightarrow$ $\boxed{HA:\ y=-4.}$

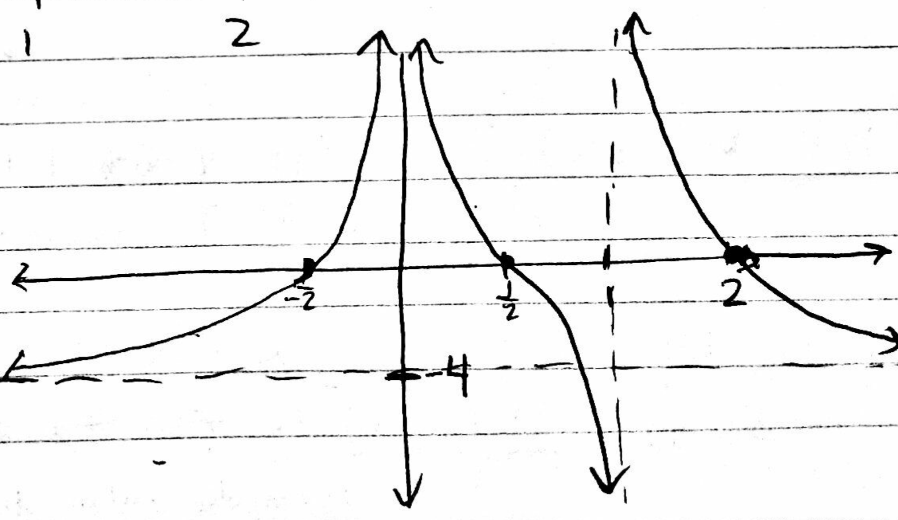
$4x^3-8x^2-x+2 = 4x^2(x-2)-1(x-2) = (4x^2-1)(x-2) = (2x-1)(2x+1)(x-2)$

Zeros: $-\tfrac{1}{2}, \tfrac{1}{2}, 2$ all mult 1

vertical asymptotes $\longrightarrow$ poles: 0 (mult 2), 1 (mult 1)

$g(-1) = \dfrac{-4-8+1+2}{1(2)} = \dfrac{-9}{2}$





Problem 7(c): $h(x) = \dfrac{10(x^4-16)}{5(x+2)^3} = \dfrac{10(x^2+4)(x^2-4)}{5(x+2)^3} = \dfrac{10(x-2)(x+2)(x^2+4)}{5(x+2)^3}$

$= \dfrac{10(x-2)(x^2+4)}{5(x+2)^2}$ if $x \neq -2$ (undef'd if $x=-2$).

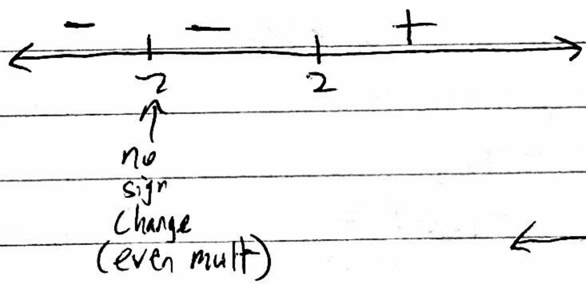
deg(top) = deg(bottom) + 1. $\Rightarrow$ slant asy, found by long div.

$5(x+2)^3 = 5(x^3+6x^2+12x+8) = 5x^3+30x^2+60x+40.$

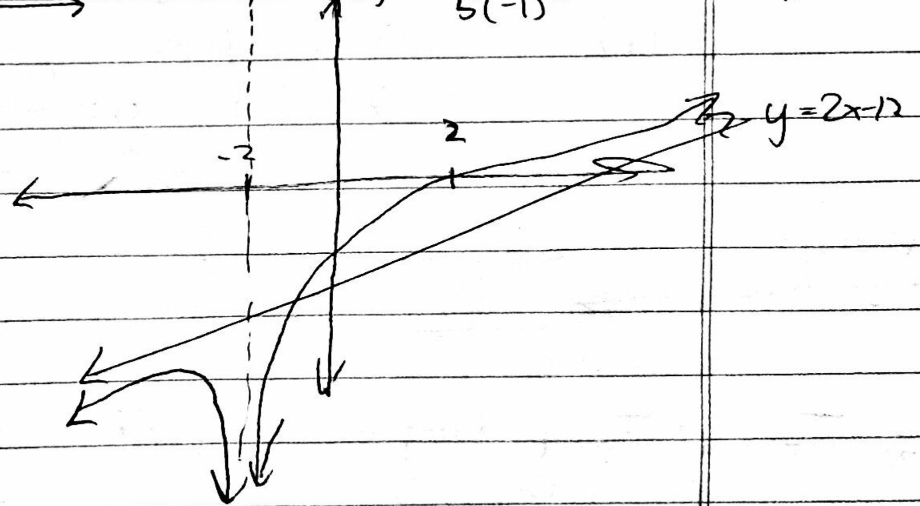
$$\begin{array}{r l}
 & 2x - 12 \\
5x^3+30x^2+60x+40 \,\big) & 10x^4+0x^3+0x^2+0x-160 \\
 & -(10x^4+60x^3+120x^2+80x) \\
 & -60x^3-120x^2-80x-160 \\
 & -(-60x^3-360x^2-720x-480) \\
 & 240x^2+640x+320
\end{array}$$

$\Rightarrow$ Slant asy $\boxed{y = 2x-12}$

Zero @ $x=2$ (mult 1), pole @ $x=-2$ (mult 2)



$h(-3) = \dfrac{10(-5)(13)}{5(-1)^2} = -\#$



Problem 8. $f(0) = 2$, $f(1) = 4-8-1+2 = -3$

sign change $\Rightarrow$ IVT guarantees a root

Problem 10: Let $f(x) = x^7+3x^4+2$. $(4x+1)$ a factor $\Leftrightarrow -\frac{1}{4}$ a zero

$f(-\frac{1}{4}) = \underbrace{(-\tfrac{1}{4})^7+3(-\tfrac{1}{4})^4+2}_{\text{remainder when dividing } f \text{ by } 4x+1 \text{ (remainder thm)}} \neq 0 \Rightarrow (4x+1)$ not a factor get from Conjugate roots thm

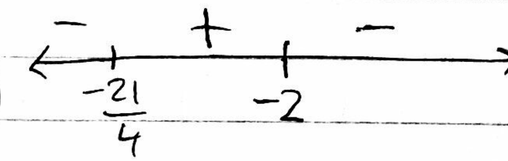
Problem 11: $(x-3)^2[x-(1-5i)][x-(1+5i)][x-(2+4\sqrt{3})][x-(2-4\sqrt{3})]$

Problem 12: (a). $\frac{3x-7}{x+2} \le 7$

$\frac{3x-7}{x+2} - 7\cdot\frac{x+2}{x+2} \le 0$

$\frac{3x-7-7(x+2)}{x+2} \le 0$

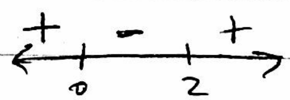
$\frac{-4x-21}{x+2} \le 0$



$\boxed{\left[-\frac{21}{4}, -2\right)}$

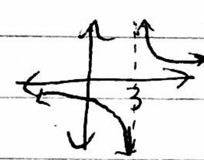
(b). no solution

(c). $\frac{x}{x-2} > \frac{2x}{7x-14} \Rightarrow \frac{7}{7}\cdot\frac{x}{x-2} - \frac{2x}{7x-14} > 0 \Rightarrow \frac{5x}{7(x-2)} > 0$



$\boxed{(-\infty, 0) \cup (2, \infty)}$

4.1 #48: (a).

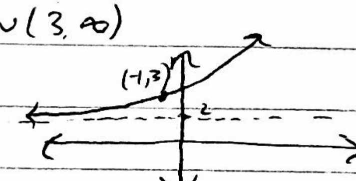


(b). s is one-to-one. $x = \frac{2}{y-3} \Rightarrow xy - 3x = 2 \Rightarrow xy = 3x+2 \Rightarrow \boxed{s^{-1}(x) = y = \frac{3x+2}{x}}$

(c). $\text{dom}(s^{-1}) = \text{range}(s) = (-\infty, 0) \cup (0, \infty)$

$\text{range}(s^{-1}) = \text{domain}(s) = (-\infty, 3) \cup (3, \infty)$

4.2 #28: (a). 4^x shifted left 1 & up 2.



(b). domain: all real #s, range: $(2, \infty)$

(c). $x = 4^{y+1} + 2 \Rightarrow x - 2 = 4^{y+1} \Rightarrow \boxed{y = \log_4(x-2) - 1}$

Problem 13: $w(z(x)) = \frac{6}{\left(\frac{6-2x}{x}\right) + 2\cdot\frac{x}{x}} = \frac{6}{\left(\frac{6}{x}\right)} = x$

$z(w(x)) = \frac{x+2}{x+2}\cdot\frac{6 - 2\left(\frac{6}{x+2}\right)}{\left(\frac{6}{x+2}\right)} = \frac{\frac{6x+12-12}{(x+2)}}{\left(\frac{6}{x+2}\right)} = x.$ They're inverses.

Problem 14: (a) not a fn, not one-to-one ; (b). one-to-one but not a function;

(c). one-to-one function.

4.2, #46:

(a). $8000\left(1+\frac{.035}{12}\right)^{12\cdot 20}$

(b). $8000\, e^{(.035)(20)}$

(c). $8100 = 8000\, e^{.035t} \Rightarrow t = \dfrac{\ln\left(\frac{8100}{8000}\right)}{.035}$

Problem 15:

(a). $\log_{16}\left(\frac{1}{4}\right) = -\frac{1}{2}$

(b). $5-2=3$

(c). 0

(d). Typo: $(-3+3\sqrt{3}\,i)^{10} = \left[6\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right]^{10}$

$= \left[6 \operatorname{cis}\left(\frac{2\pi}{3}\right)\right]^{10} = 6^{10}\operatorname{cis}\left(\frac{20\pi}{3}\right) = 6^{10}\operatorname{cis}($

$= 6^{10}\left[-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]$

(e). $2\left(-\frac{\pi}{4}\right) - \left(-\frac{\pi}{6}\right) + \left(\frac{5\pi}{6}\right) = \frac{2\pi}{3} - \frac{\pi}{2} = \boxed{\frac{\pi}{6}}$

(f). $\boxed{-\frac{\pi}{3}}$

(g). $\theta = \sin^{-1}\frac{4}{5} \Rightarrow \sin\theta = \frac{4}{5}$ $\quad \cos 2\theta = 1-2\sin^2\theta = 1-2\left(\frac{4}{5}\right)^2 = \boxed{\frac{17}{25}}$

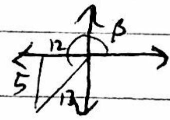
Problem 16: $\frac{\pi}{3}, \frac{2\pi}{3}$

Problem 17: $\sin^{-1}\left(-\frac{1}{5}\right)$ & $\pi + \left(-\sin^{-1}\left(-\frac{1}{5}\right)\right)$ ← simpler form: $\pi + \sin^{-1}\left(\frac{1}{5}\right)$

5.3, #61: $\frac{5\pi}{6}, \frac{11\pi}{6}$

Problem 18:

$\Rightarrow \tan\alpha = -\frac{3}{4}$



$\Rightarrow \tan\beta = \frac{5}{12}$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{-\frac{3}{4}+\frac{5}{12}}{1+\frac{3}{4}\cdot\frac{5}{12}}$

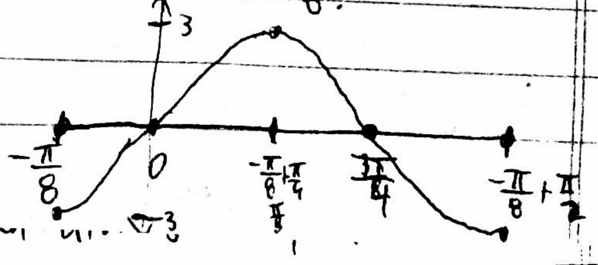
$\frac{3\pi}{2} < \alpha < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{\alpha}{2} < \pi \Rightarrow \frac{\alpha}{2}$ in QII & $\cos\frac{\alpha}{2} < 0$

$$\boxed{\cos\frac{\alpha}{2} = -\sqrt{\frac{1+\left(\frac{4}{5}\right)}{2}}}$$

Problem 19: unit circle: $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ ←

unit vector: $\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle$ ← Same Components

Problem 20: period: $\frac{2\pi}{4} = \frac{\pi}{2}$; phase shift: $-\frac{\left(\frac{\pi}{2}\right)}{4} = -\frac{\pi}{8}$.

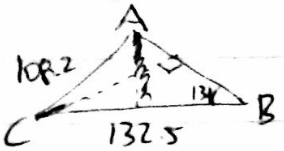


Problem 21 $Q(t) = 125\left(\frac{1}{2}\right)^{t/1620}$

Problem 22 see Review session notes.

Problem 23.

(a).



$\Rightarrow$ SSA case, ambiguous & law of sines

$h = 132.5 \sin(13.1^\circ) \approx 30.03$

$30.03 \approx h < 108.2 < 132.5$

$\Downarrow$

2 Δs! (A acute & A obtuse)

$$\frac{\sin A}{132.5} = \frac{\sin 13.1^\circ}{108.2} \Rightarrow A \approx 16.11^\circ \text{ or } 180 - 16.11 = 163.89^\circ$$

Case 1 $A \approx 16.11^\circ$ $\Rightarrow C = 180 - 16.11^\circ - 13.1^\circ = 150.79^\circ$

$$\frac{\sin 16.11^\circ}{132.5} = \frac{\sin 150.79^\circ}{c} \Rightarrow \boxed{c = 233.06}$$

Case 2 $A \approx 163.89^\circ \Rightarrow C \approx 3.01^\circ$

$$\frac{\sin 3.01^\circ}{c} = \frac{\sin 163.89}{132.5} \Rightarrow \boxed{c = 25.07}$$

(b). SSS $\Rightarrow$ law of cosines

$$9.7^2 = 2.3^2 + 10.8^2 - 2(2.3)(10.8)\cos C$$

$$\Rightarrow \cos C = \frac{9.7^2 - 2.3^2 - 10.8^2}{-2(2.3)(10.8)} \Rightarrow \boxed{C \approx 55.9^\circ}$$

$\approx \frac{-27.6}{49.68}$

$$\frac{\sin 55.9^\circ}{9.7} = \frac{\sin A}{2.3} \Rightarrow \boxed{A \approx 11.3^\circ}$$

$$\frac{\sin 55.9}{9.7} = \frac{\sin B}{10.8} \Rightarrow \boxed{B \approx 67.2^\circ}$$

I made an error.

Problem 24. (a) $\log_4 x = 3 \Rightarrow x = 4^3 = 64$.

(b). Get all in terms of $\cos\theta$

$\cos\theta\cos2\theta - \sin\theta\sin2\theta - \cos\theta = \sin2\theta$

$\cos\theta(2\cos^2\theta - 1) - \sin\theta(2\sin\theta\cos\theta) - \cos\theta = 2\sin\theta\cos\theta$

$2\cos^3\theta - \cos\theta - 2(\sin^2\theta)\cos\theta - \cos\theta = 2\sin\theta\cos\theta$

$2\cos^3\theta - 2\cos\theta - 2(1-\cos^2\theta)\cos\theta = 2\sin\theta$

$4\cos^3\theta - 4\cos\theta = 2\sin\theta$

SKIP

Problem 24: (ctd)

(c). $(6\cos x + 1)(\cos x - 1) = 0$

$\Rightarrow \cos x = -\frac{1}{6}$ or $\cos x = 1$

$$\boxed{x = \begin{cases} \cos^{-1}\left(-\frac{1}{6}\right) + 2k\pi \\ \pi + \cos^{-1}\left(\frac{1}{6}\right) + 2k\pi \\ 2k\pi \end{cases}}$$

(d). Get all in terms of $\cos 2x$.

$$(2\cos^2 2x - 1) - 3\cos 2x - 1 = 0$$

$$2\cos^2 2x - 3\cos 2x - 2 = 0$$

$$(2\cos 2x + 1)(\cos 2x - 2) = 0$$

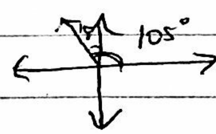
$\Rightarrow \cos 2x = -\frac{1}{2}$ or $\cancel{\cos 2x = 2}$ impossible

$$2x = \begin{cases} \frac{2\pi}{3} + 2k\pi \\ \frac{4\pi}{3} + 2k\pi \end{cases} \Rightarrow \boxed{x = \begin{cases} \frac{\pi}{3} + k\pi \\ \frac{2\pi}{3} + k\pi \end{cases}}$$

(on $[0, 2\pi)$: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$)

Problem 25: $\frac{2-2i}{3+3i} = \frac{2\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)}{3\sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)} = \frac{2\sqrt{2}\left(\operatorname{cis}\frac{7\pi}{4}\right)}{3\sqrt{2}\left(\operatorname{cis}\frac{\pi}{4}\right)} = \frac{2}{3}\operatorname{cis}\frac{3\pi}{2} = \boxed{-\frac{2}{3}i}$

Problem 26:



$$\boxed{\vec{F} = 300\langle \cos 105°, \sin 105° \rangle}$$

Want $\vec{F}_1$ s.t. $\vec{F}_1 + \vec{F} + \langle 3, 7\rangle = \vec{0} \Rightarrow \boxed{\vec{F}_1 = \langle -3, -7\rangle + \langle -300\cos 105°, -300\sin 105°\rangle}$

Problem 27: $\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v} = \frac{-11}{13}\langle 3, 2\rangle$

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{-11}{\sqrt{74}\sqrt{13}}\right)}$

Problem 28: Eqn 3 = 3 · Eqn 1, so Eqn 3 is a duplicate.
Have 2 eqns in 3 unknowns, so
either no soln or ∞ many solns!

1 less eqn than # of unknowns $\Rightarrow$ 1 free variable.

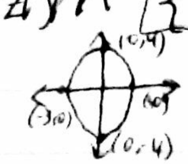
$$\begin{array}{r} -3x + 4y - z = -4 \\ + \quad x + 2y + z = 4 \\ \hline -2x + 6y \quad = 0 \end{array} \Rightarrow 2x - 6y \Rightarrow x = 3y.$$

$$[\,3y + 2y + z = 4 \quad \& \quad -3(3y) + 4y - z = -4\,]$$

$$5y + z = 4 \Rightarrow z = 4 - 5y.$$

solns are $\boxed{\{(3y, y, 4-5y) : y \text{ any real } \#\}}$

Problem 29: $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$



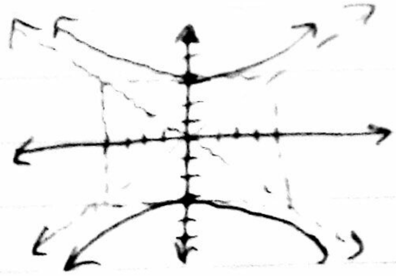
$\leftarrow$ y axis major

$a = 4,\ b = 3,\quad c^2 = a^2 - b^2 = 16 - 9 \Rightarrow c = \sqrt{7}. \Rightarrow e = \dfrac{c}{a} = \boxed{\dfrac{\sqrt{7}}{4}}$

& $\boxed{\text{foci @ } (0, \pm\sqrt{7})}$

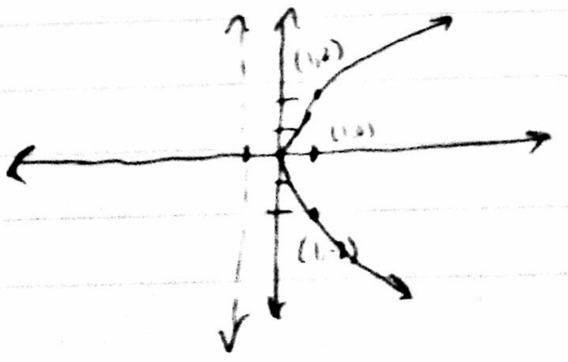
Problem 30: $\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1 \Rightarrow$ opens vertically.

$c^2 = a^2 + b^2 = 25 \Rightarrow c = 5 \Rightarrow \boxed{\text{foci @ } (0, \pm 5)},\ \boxed{\text{eccentricity} = \frac{5}{3}}$



$\rightsquigarrow$ asys $\boxed{y = \pm \frac{3}{4} x}$

Problem 31: $y^2 = 4x = 4px \Rightarrow p = 1. \Rightarrow$ ① opens right.
② focus @ $(1,0)$ & directrix $x = -1$.



$\Rightarrow$ endpts of latus rectum: $(1, \pm 2)$
focal diameter 4

Problem 32: (a) foci @ $(0, \pm 5) \Rightarrow c = 5$. $e = \frac{c}{a} = \frac{5}{4} \Rightarrow a = 4$.

$\Rightarrow$ opens vertically, so a under y^2.

$c^2 = a^2 + b^2 \Rightarrow 5^2 = 4^2 + b^2 \Rightarrow b = 3$.

$$\boxed{\frac{y^2}{16} - \frac{x^2}{9} = 1}$$

(b). $a = 5$, $c = 4$, $c^2 = a^2 - b^2 \Rightarrow b = 3$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

(c). directrix @ $x = 3 \Rightarrow p = \pm 3$ & opens left

$\Rightarrow p = -3$.

$y^2 = 4px \Rightarrow \boxed{y^2 = -12x}$