Problem 1, Form B:

(a). We're taking $g(x) = (x+2)^2$ for x between -5 and 0. $g(x)$ has a zero at -2, i.e. $g(-2) = 0$. Moreover, $(x+2)^2$ is ALWAYS ≥ 0, so the only choice we're left with is (i). $[0, 9]$.

(b). We need to determine whether $P(1)$ and $P(4)$ have opposite signs. Indeed, $P(1) = 1 - 7 + 8 = 2$, and $P(4) = 4 \cdot 4^2 - 7 \cdot 4^2 + 8 = (-3) \cdot 4^2 + 8 = -40$. Thus, YES, the IVT applies.

(c). We need the vertex (h, k), so then we'll have the standard/vertex form $f(x) = a(x-h)^2 + k$, where a is the leading coefficient. We know that $h = -\frac{b}{2a} = -\frac{-14}{2 \cdot 7} = 1$, and $k = f(h) = f(1) = 7 - 14 + 11 = 4$, so then our vertex form is $f(x) = 7(x-1)^2 + 4$.

(d). We need to verify whether $f(g(x)) = x$ and $g(f(x)) = x$. $f(g(x)) = 8(g(x) - 5) = 8\left(\frac{x+5}{8} - 5\right) = 8\frac{x+5}{8} - 5 \cdot 8 = x + 5 - 40 = x - 35 \neq x$. Thus, they are NOT inverses.

(e). Make a sign chart. Our number line is split up with $x = 4$ and $x = 7$. So, we need to consider $x < 4$, $4 < x < 7$, and $x > 7$. If $x < 4$, $4 - x > 0$ and $x - 7 < 0$, so then $\frac{4-x}{x-7} < 0$ and we include $x < 4$ in our answer. Also, if we plug in 4, we get 0, so we include 4. Next, if $4 < x < 7$, then $4 - x < 0$ and $x - 7 < 0$, so then $\frac{4-x}{x-7} > 0$, so we exclude $(4, 7)$ from our answer. Finally, if $x > 7$, $4 - x < 0$ and $x - 7 > 0$, so then $\frac{4-x}{x-7} < 0$, so we include $(7, \infty)$ in our answer. However, we can't include $x = 7$ because the rational function isn't defined at $x = 7$. Thus, our final answer is $(-\infty, 4] \cup (7, \infty)$.

Problem 2, Form A:

(a). $P(x)$ is a degree 4 (even degree) polynomial with leading coefficient $-\frac{1}{9} < 0$. Thus, as $x \to -\infty$, $P(x) \to -\infty$, and as $x \to \infty$, $P(x) \to -\infty$.

(b). $(x-5)^2$ is a factor, so 5 must be a zero of multiplicity 2 by the Factor Theorem. Also, $(x+1)^2$ is a factor, so -1 must be a zero of multiplicity 2 by the Factor Theorem.

(c). Graph the function $y = P(x)$ on your calculator in the window $[-10, 10] \times [-10, 10]$. Press the "2ND" key and then press the "TRACE" button. Select "maximum". Follow the graph to the left of the first hump, and then press ENTER. Then, go to the right of it and press ENTER. You'll then be given $X =$ and $Y =$ on the bottom line of your calculator. Your relative maximum is the y-value. If it's not exactly zero (e.g. "-2.69E-12," which means -2.69×10^{-12}), then it will be VERY close to 0, so write $y = 0$, occurring at $x = -1$. Do the same with the other hump, and it will also be $y = 0$ at $x = 5$. The relative minimum will take place in the valley just to the right of the y-axis. To find it, press the "2ND" key and then press the "TRACE" button. Select "minimum". Follow the graph to the left of the dip's lowest point, and then press ENTER. Then, go to the right of it and press ENTER again. You'll then be given $X =$ and $Y =$ on the bottom line of your calculator. Your relative minimum is the y-value, which should either be 9 or VERY CLOSE to 9, and that will occur at $x = 2$.

(d). $P(x)$ is increasing on intervals before a relative max and/or after a relative min. Thus, P is increasing on $(-\infty, -1) \cup (2, 5)$ [first interval is before the first relative max, and the second interval is just after the relative minimum and just before second relative maximum]. Also, $P(x)$ decreases on the intervals just before a relative minimum or just after a relative maximum. Thus, P is decreasing on $(-1, 2) \cup (5, \infty)$ [first interval just after the relative max and before the relative min, and the second interval just after the second relative max].

Problem 3, Form B:

(a). By the Factor Theorem, $x - 1$ is a factor of the polynomial IF AND ONLY IF 1 is a zero of the polynomial. If we call our polynomial $f(x) = x^{100} - 3x + 2$, then $f(1) = 1^{100} - 3 \cdot 1 + 2 = 1 - 3 + 2 = 0$, so 1 is a zero of $f(x)$ and by the Factor Theorem $(x-1)$ is a factor of $f(x)$.

(b). The average rate of change of a function $f(x)$ on the interval $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$. So, since $a = -2$ and $b = 1$, our average rate of change is $\frac{f(1)-f(-2)}{1-(-2)} = \frac{11-[9(-2)^5+2\cdot(-2)]}{1+2} = \frac{11+292}{3} = \frac{303}{3} = 101$.

(c). We use the Factor Theorem and the Conjugate Zero Theorem. Since 9 is a zero of multiplicity 4, $(x-9)^4$ is a factor. Since $4+5i$ is a zero, its conjugate, $4-5i$ must also be a zero. The Factor Theorem then tells us that $[x-(4+5i)]$ and $[x-(4-5i)]$ must be factors. Thus, putting it all together, we know that the product $(x-9)^4[x-(4+5i)][x-(4-5i)]$ must be a factor of the polynomial $P(x)$. Since this product has degree 6 and the polynomial P has degree 6, this product satisfies all the properties we need P to satisfy, so we can just let $P = (x-9)^4[x-(4+5i)][x-(4-5i)]$.

(d). Vertical asymptotes occur when the denominator is zero. $x^2-2x-3 = (x-3)(x+1)$, so $x^2-2x-3 = 0$ when $(x-3)(x+1) = 0$, and by the zero product property, this means the denominator is zero when $x = 3$ and when $x = -1$. Thus, our vertical asymptotes are at $x = 3$ and $x = -1$. Next, horizontal asymptotes occur in one of two cases: (1) when the degree of the numerator and denominator are equal, in which case the horizontal asymptote is $y = c/d$ where c is the leading coefficient of the top and d is the leading coefficient of the bottom; (2) when the degree of the bottom exceeds the degree of the top, in which case, the horizontal asymptote is $y = 0$. Thus, there are NO horizontal asymptotes because neither of these conditions are met. There is a slant asymptote if the degree of the top is one larger than the degree of the bottom. Indeed, the degree of the top is 3 and the degree of the bottom is 2, so there MUST be a slant asymptote, found by long division (divide the top by the bottom; disregard the remainder). Following through all the way with the long division, we get $y = 4x + 8$ as our slant asymptote.

Problem 4, Form A:

(a). Shrinking horizontally by a factor of 2 means plugging 2x in place of x into the original equation, so we're considering $h(2x)$. Next, shifting up 3 means adding 3 to that, so we have $h(2x)+3$. Finally, reflecting in/over the x-axis means replacing our function with its negative, so the result we want is $-[h(2x) + 3] = -[(2x)^3 - 8] + 3 = -[8x^3 - 5] = -8x^3 + 5$.

(b). (i). $(G \circ F)(x) = \frac{3}{(F(x))^2} = \frac{3}{(\sqrt{x+1})^2} = \frac{3}{x+1}$. The domain of this final result would be $x \neq -1$, BUT we have to restrict to the domain of our original function $F(x)$ we're plugging into G too. The domain of $F(x)$ is $x \geq -1$. Thus, we need $x \geq -1$ AND $x \neq -1$, so our domain is $(-1, \infty)$.

(b). (ii). The domain of $\frac{F}{G}$ is ALWAYS all x in BOTH the domain of F AND the domain of G where $G(x) \neq 0$. The domain of F is $[-1, \infty)$, the domain of G is $x \neq 0$, and $G(x)$ is NEVER zero (you can see this with the graph of G: $y = 0$ is a horizontal asymptote). Thus, our domain is $[-1, 0) \cup (0, \infty)$.

(b). (iii). We want to find a function $f(x)$ where $G = f \circ g$, where $G(x) = \frac{3}{x^2}$ and $g(x) = x - 7$. The first thing we notice is that our target function G involves x, not $x - 7$. So, we'd like to make all instances of $x - 7$ be x instead. We do that by adding 7; let's call the function that does that n, in which case $n(x) = x + 7$ is our formula. Then, $n(g(x)) = (x - 7) + 7 = x$, and we want to get to $G(x) = \frac{3}{x^2}$. So, $G(x) = G(n(g(x)) = [(G \circ n) \circ g](x)$, and our desired $f(x)$ is $(G \circ n)(x) = \frac{3}{(n(x))^2} = \frac{3}{(x+7)^2}$.

Problem 5, Form B:

(a). We have a degree 3 polynomial with all four terms. This should suggest we factor by grouping. Group the first two terms together and pull out a common factor from that grouping, then group the last two terms together and pull out their common factor. $Q(x) = x^3+3x^2+25x+75 = (x^3+3x)+(25x+75) = x^2(x+3)+25(x+3)$. We then notice that we have something that looks like $ab+cb$, which we can factor as $(a+c)b$. Thus, we have $Q(x) = (x^2+25)(x+3)$. The polynomial x^2+25 is ordinarily an irreducible polynomial, BUT the problem asks us to factor completely into linear factors (i.e. factors that look like $ax+b$) which may involve complex numbers. The zeros of x^2+25 are $\pm 5i$, so that means x^2+25 factors into $(x+5i)(x-5i)$. So, our final factorization is $Q(x) = (x-5i)(x+5i)(x+3)$.

(b). We first notice we have a difference of squares. $x^4 - 81 = (x^2)^2 - 9^2$. Thus, $P(x) = (x^2+9)(x^2-9)$. $x^2 - 9$ is a difference of squares too, so $x^2 - 9 = (x-3)(x+3)$. Also, since we need to decompose into linear factors and the roots of $x^2 + 9$ are $\pm 3i$, we have that $P(x) = (x-3i)(x+3i)(x-3)(x+3)$.

Problem 6, Form A:

We solve these problems by switching x and y and then solving for y. So, we take $y = \frac{5x+9}{6x-1}$ and replace it with $x = \frac{5y+9}{6y-1}$. If we want to solve for y, we better clear the denominator, so we do that to get $x(6y-1) = 5y+9$. Next, we distribute: $6xy - x = 5y + 9$. To solve for y in this equation, we MUST get everything involving y onto one side and then get everything else to the other side. So, we want $6xy - 5y = x + 9$. Next, treating x like a constant and y as the variable, to solve for y, we must factor out the common y: $(6x-5)y = x+9$. From there, to isolate y, we divide both sides by $6x - 5$. Thus, $y = \frac{x+9}{6x-5}$. We have y in terms of JUST x, so we're done, and $f^{-1}(x) = \frac{x+9}{6x-5}$.