Fact 0.1 **[Horizontal Asymptotes]** *Let* $f(x) = \frac{p(x)}{q(x)}$ *be a* **rational function** *(quotient of polynomials).*

(i) *If* $\deg p > \deg q$*, there are no horizontal asymptotes.*

(ii) *If* $\deg p = \deg q$*,* a_n *is the leading coefficient of* $p(x)$*, and* b_m *is the leading coefficient of* $q(x)$*, then* f *has a horizontal asymptote* $y = \frac{a_n}{b_m}$*.*

(iii) *If* $\deg p < \deg q$*, then* $y = 0$ *is the horizontal asymptote for* f*.*

Fact 0.2 **[Vertical Asymptotes and Holes]** *Let* $f(x) = \frac{p(x)}{q(x)}$ *be a rational function. The vertical asymptotes of* $f(x)$ *are* $x = c$*, where* c *is a zero for* $q(x)$ *but NOT a zero for* $p(x)$*. If* $p(x)$ *and* $q(x)$ *have a common zero at* $x = c$*, then there will be a hole in the graph of* $f(x)$ *at* $x = c$*.*

Fact 0.3 **[Slant Asymptotes]** *Let* $f(x) = \frac{p(x)}{q(x)}$ *be a rational function, and suppose that* $\deg p = \deg q + 1$*. Then,* f *has a slant asymptote* $y = ax + b$*, where* $p(x) = [ax + b]q(x) + r(x)$*, where* $r(x)$ *is the remainder when* $p(x)$ *is divided by* $q(x)$ *with long division and* $(ax + b)$ *is obtained by long division.*

Procedure 0.4 **[Graphing Rational Functions]** *Find the* x *and* y *intercepts. Next, find all asymptotes and determine whether any horizontal or slant asymptotes were crossed. Then, determine the end behavior. Finally, use the x-values for your intercepts, vertical asymptotes, and points where slant and horizontal asymptotes were crossed to divide the number line into pieces, then plot a test points, with at least one point* (x, y) *with an* x *in each of these pieces of the number line.*

3.5, #19,22: Determine the vertical asymptotes for the following functions.

(a). $h(x) = \frac{x-3}{2x^2-9x-5}$.

(b). $n(x) = \frac{6}{x^4+1}$.

3.5, #44,46: Determine the asymptotes for the following functions.

(a). $l(x) = \frac{x^3+3x^2-2x-4}{x^2-7}$

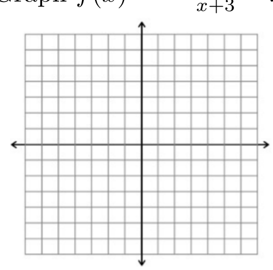
(b). $m(x) = \frac{3x-4}{x^3+2x^2-9x-18}$

3.5, #30,32: Determine the horizontal asymptotes for the following functions and the point(s) where they are crossed (if any).

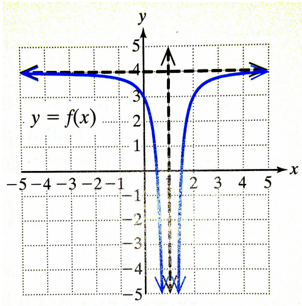
(a). $q(x) = \frac{8}{x^2+4x+4}$.

(b). $r(x) = \frac{-4x^2+5x-1}{x^2+2}$.

3.5, #83: Graph $f(x) = \frac{x^2+7x+10}{x+3}$.



3.5, #16: Use the picture below to fill in the blanks.



(a). As $x \to -\infty$, $f(x) \to$ ______.

(b). As $x \to 1^-$, $f(x) \to$ ______.

(c). As $x \to 1^+$, $f(x) \to$ ______.

(d). As $x \to \infty$, $f(x) \to$ ______.

(e). The graph is increasing over the interval(s) ______.

(f). The graph is decreasing over the interval(s) ______.

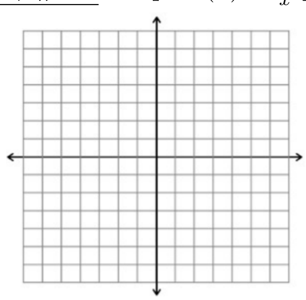
(g). The domain is ______.

(h). The range is ______.

(i). There is a vertical asymptote at ______.

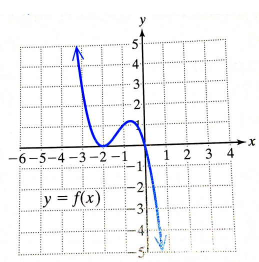
(j). There is a horizontal asymptote at ______.

3.5, #89: Graph $v(x) = \frac{2x^4}{x^2+9}$.



3.6, #12: Use the graph below to determine the intervals on which (a) $f(x) < 0$, (b) $f(x) > 0$, (c) $f(x) \leq 0$, (d) $f(x) \geq 0$.

12.



3.6, #17: Solve the following equalities and inequalities.
(a). $-x^2 + x + 12 = 0$.
(b). $-x^2 + x + 12 < 0$.
(c). $-x^2 + x + 12 > 0$.
(d). $-x^2 + x + 12 \leq 0$.
(e). $-x^2 + x + 12 \geq 0$.

3.6, #19: Solve the following equalities and inequalities.
(a). $a^2 + 12a + 36 = 0$.
(b). $a^2 + 12a + 36 < 0$.
(c). $a^2 + 12a + 36 > 0$.
(d). $a^2 + 12a + 36 \leq 0$.
(e). $a^2 + 12a + 36 \geq 0$.

3.6, #23,31,40,48: Solve the following inequalities.
(a). $3w^2 + w < 2(w + 2)$.

(b). $16p^2 \geq 2$.

(c). $3x^3 - 3x < 4x^2 - 4$.

(d). $(4x + 1)^2 > -6$.

3.6, #62,68,72,82: Solve the following inequalities.
(a). $\frac{5-x}{x+1} \geq 0$.

(b). $\frac{p^2-2p-8}{p-1} \geq 0$.

(c). $\frac{3x}{3x-7} \leq 1$.

(d). $\frac{5}{2-x} \leq \frac{3}{3-x}$.