

Theorem 0.1 [Remainder Theorem] *If a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$; that is, $f(x) = [x - c]q(x) + f(c)$ for some polynomial $q(x)$.*

Theorem 0.2 [Factor Theorem] *Let $f(x)$ be a polynomial with real coefficients. Then, $f(c) = 0$ if and only if $(x - c)$ is a factor of $f(x)$.*

Theorem 0.3 [Rational Zero Theorem] *Consider a polynomial $f(x)$ with integer coefficients. Then, if $f(x)$ has rational (fraction) zeroes, they must be of the form $\pm \frac{p}{q}$ where p divides the constant term and q divides the leading coefficient of f .*

Theorem 0.4 [Conjugate Zeroes Theorem] *Consider a polynomial $f(x)$ with integer coefficients. If $a + bi$ is a zero for $f(x)$, then $a - bi$ must be too. Also, if $a + b\sqrt{c}$ is a zero for $f(x)$, then $a - b\sqrt{c}$ must be too.*

Theorem 0.5 [Fundamental Theorem of Algebra] *Consider a degree n polynomial $f(x)$ with complex coefficients. Then, $f(x)$ has n complex zeroes (not necessarily distinct; we count multiplicity), and $f(x)$ may be written as a product of n linear factors with complex coefficients.*

3.3, #13: Use long division to find
 $(-20x^2 + 6x^4 - 16) \div (2x + 4)$

3.3, #41: Given $f(x) = 2x^4 - 5x^3 + x^2 - 7$, evaluate $f(4)$ and determine the remainder when $f(x)$ is divided by $(x - 4)$.

3.3, #17: Use long division to find

$$\frac{6x^4 + 3x^3 - 7x^2 + 6x - 5}{2x^2 + x - 3}$$

3.3, #55: Use the Factor Theorem to determine whether $(x + 5)$ and $(x - 2)$ are factors of $f(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$.

3.3, #77: Find a polynomial of minimal degree with integer coefficients that has roots -2 (multiplicity 2) and $-3i$.

3.4, #33: Given $2 - 5i$ is a zero for the polynomial $f(x) = x^4 - 4x^3 + 22x^2 + 28x - 203$, find all zeros of f and write $f(x)$ as a product of linear factors.

3.3, #91: Find m so that $x + 2$ is a factor of $4x^3 + 5x^2 + mx + 2$.

3.4, #37: Given $-3 + 2i$ and $-\frac{1}{4}$ are zeros for the polynomial $f(x) = 4x^5 + 37x^4 + 117x^3 + 87x^2 - 193x - 52$, find all zeros of f and write $f(x)$ as a product of linear factors.

3.4, #95: Explain why a degree 3 polynomial with real coefficients MUST have at least one real zero.

3.4, Example (Like # 93): Determine whether the Intermediate Value Theorem guarantees that $f(x) = 4x^3 - 5x^2 - 23x + 6$ has a zero on the interval $[0, 1]$. If there is a zero, find it using the Rational Zeroes Theorem and the Factor Theorem. Then, find the remaining zeroes of $f(x)$ and factor $f(x)$ into a product of linear factors.