Math 2153 - Spring 2017 Name: ______________________

Quiz 6 - Take Home (10 pts) Recitation Time: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, give me EXACT answers (do NOT use your decimals in your final answers, though they may be used to approximate where a number is in a graph).

Problem 1 [2 pts] Express the volume of the solid bounded between the surface $z = xy^2\cos(xy^3)$ and the rectangle $R = \{(x,y) : 0 \le x \le \frac{\pi}{2}, 0 \le y \le 1\}$ in the xy-plane as an integral. Then, evaluate that integral.

Problem 2 [3 pts] In this problem, we'll find the closest point(s) on the elliptic cone $4x^2 + 9y^2 - 16z^2 = 0$ to the point $(1,1,0)$.

(a). [0.5 pts] Set up the three equations resulting from using the method of Lagrange multipliers for the above scenario.

(b). [1.5 pts] Use one of the above equations and the zero product property from high school algebra to find a potential value for λ and use that to solve for x, y, and z. Also, consider the other case that comes from that application of the zero-product property and find what point on the cone arises from it.

(c). [1 pt] Use (b) to find the closest point(s) on the elliptic cone $4x^2 + 9y^2 - 16z^2 = 0$ to the point $(1,1,0)$. Justify how you know that the point you found is indeed the closest point. You are encouraged to specifically cite theorems on the recitation handouts and appeal to basic geometric intuition about the cone.

Problem 3 [5 pts] In this problem, we'll find the absolute maximum and absolute minimum values of the function $f(x, y) = 8xy$ in/on the region $R = \{(x, y) : 4x^2 + 9y^2 \leq 36\}$.

(a). [1 pt] Find the critical point(s) of f in the interior of R, classify it/them with the 2nd Derivative Test, and evaluate f at it/them.

(b). [0.5 pts] Set up the pair of equations obtained from using Lagrange multipliers on the boundary.

(c). [0.5 pts] Use part (b) to eliminate λ, and then use this elimination to obtain an equation $p(x, y) = 0$ where $p(x, y)$ is a polynomial.

(d). [0.5 pts] Factor the above polynomial and use the zero product property from high school algebra to determine two possible SIMPLE relations between x and y.

(e). [1.5 pts] Plug the relations found in part (d) into the constraint (boundary curve) to find the pairs (x, y) on the boundary curve where extreme values occur (this comes for free from Lagrange Multipliers). Then, evaluate f at those points.

(f). [1 pt] Use the information from parts (a) and (e) and two theorems and/or procedures from the recitation handouts (state which theorems they are and WHY they apply) to determine the absolute minimum and maximum values of f in/on R.