## Math 2153 S2017: Midterm remarks and answers

**Problem 1:**

(a). Area $= \frac{\sqrt{6}}{2}$

(b). $2x + y - z = 3$.

(c). $\mathbf{r}(t) = \langle 1+t, 2, 1+2t \rangle$

---

**Problem 2:**

(1). $\mathbf{u} \cdot \mathbf{v} = 5$; $\theta = \frac{\pi}{3}$; $\text{scal}_{\mathbf{v}}\mathbf{u} = \frac{5}{\sqrt{10}}$; $\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{1}{2}\langle 2, 1, \sqrt{5} \rangle$.

(2). $\left(\frac{1}{2}\ln 2\right)\mathbf{i}$

(3). Notice $x^2 + 2x - xy - 2y = x(x+2) - y(x+2) = (x-y)(x+2)$, so by cancelling the common factor in the fraction, the answer is 3.

(4)(a). For $h \neq 0$, $f(h,0) = \frac{0}{h^4} = 0$, so $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0}{h} = 0$.

(4)(b). $f$ is not continuous at 0 and hence not differentiable. To show $f$ is not continuous, notice that $f$ has homogeneous denominator (all terms have same degree), so by setting $y = mx$ for different values of $m$, one would expect to get different limits as $x \to 0$ (e.g. limit is 0 if $m = 0$, $1/2$ if $m = 1$, etc.).

---

**Problem 3:**

(a). In general the arc length function is $s(t) = \int_0^t |\mathbf{r}'(u)|du$ (note: a number of you set the upper bound of integration to be $\infty$, not $t$; DON'T DO THAT!). Thus, the answer is $s(t) = 5t$.

(b). No, since $|\mathbf{r}'(t)| = 5 \neq 1$.

(c). $\mathbf{N}(t) = \langle -\cos 3t, -\sin 3t, 0 \rangle$.

---

**Problem 4:**

(a). $\kappa(t) = \frac{2}{(1+4t^2)^{3/2}}$

(b). $\kappa'(t) = \frac{-24t}{(1+4t^2)^{5/2}}$. $\kappa' = 0$ when $t = 0$, $\kappa' > 0$ when $t < 0$, and $\kappa' < 0$ when $t > 0$, so $\kappa$ must maximize at $t = 0$. Therefore, at the point $\mathbf{r}(0) = (0, 1)$, the maximum curvature is attained, and that maximum curvature is $\kappa(0) = 2$.

**Problem 5:**

(a). $\frac{\partial f}{\partial y} = 2ye^{xy} + xy^2e^{xy}$; $\frac{\partial^2 f}{\partial x \partial y} = 3y^2e^{xy} + xy^3e^{xy}$.

(b). $\left.\dfrac{\partial z}{\partial s}\right|_{(r,s)=(0,1)} = 6$.

---

**Problem 6:**

(a). False. $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if the vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel.

(b). True. Level curves for $z = f(x, y)$, necessarily of the form $f(x, y) = c$ are ALWAYS curves in the $xy$-plane.

(c). False. A circle is $\mathbf{r}(t) = \langle a \cos t, a \sin t \rangle$, so $\kappa(t) = \frac{1}{a}$, meaning bigger radii results in smaller $\kappa$, the opposite of what's stated.

(d). True. The torsion of a plane curve is ALWAYS zero since the binormal vector is constant, meaning $\frac{d\mathbf{B}}{ds} = 0$.

(e). True. The degree of the numerator exceeds the degree of the denominator, so it will go to 0 faster than the denominator will, forcing $\lim_{(x,y)\to(0,0)} f(x, y) = 0$. Thus, $\lim_{(x,y)\to(0,0)} f(x, y) = 0 = f(0, 0)$, meaning $f$ is continuous at $(0, 0)$.