15.2, Problem 15: Evaluate $\int_C (x^2+y^2)ds$, where C is the circle of radius 4 centered at the origin.

Solution:
Step 1: Parametrize C. We know from Math 1172 how to parametrize a circle of radius 4 centered at the origin: the parametrization is given by $\mathbf{r}(t) = \langle x(t), y(t)\rangle = \langle 4\cos t, 4\sin t\rangle$, $0 \le t \le 2\pi$.
Step 2: Find $|\mathbf{r}'(t)|$. $\mathbf{r}'(t) = \langle -4\sin t, 4\cos t\rangle$, so $|\mathbf{r}'(t)| = \sqrt{(-4\cos t)^2 + (4\sin t)^2} = 4$, since $\sin^2 t + \cos^2 t = 1$.
Step 3: Evaluate $\int_C (x^2+y^2)ds$. In general, $\int_C f ds = \int f(x(t), y(t)) \cdot |\mathbf{r}'(t)| dt$ (with numerical integration bounds). Thus, $\int_0^{2\pi} (x^2+y^2)ds = \int_0^{2\pi} [(4\cos t)^2 + (4\sin t)^2] \cdot 4dt = \int_0^{2\pi} 16 \cdot 4dt = 128\pi$.

15.2, Problem 37: Given a vector field $\mathbf{F} = \frac{\langle x,y\rangle}{(x^2+y^2)^{3/2}}$ and a curve C parametrized by $\mathbf{r}(t) = \langle t^2, 3t^2\rangle$, $1 \le t \le 2$, evaluate $\int_C \mathbf{F} \cdot \mathbf{T} ds$.

Solution: Recall in general that given a curve C parametrized by $\mathbf{r}(t) = \langle x(t), y(t)\rangle$, $a \le t \le b$, by definition $\int_C \mathbf{F} \cdot \mathbf{T} ds = \int_a^b \mathbf{F}(x(t), y(t)) \cdot \mathbf{r}'(t) dt$. **ALSO NOTE** that this same form calculates the work achieved by moving a particle with force $\mathbf{F}$ along the curve C.

So, we compute each individual component of this formula.

$$\mathbf{F}(x(t), y(t)) = \frac{\langle t^2, 3t^2\rangle}{((t^2)^2 + (3t^2)^2)^{3/2}} = \frac{1}{10^{3/2}} \langle \frac{1}{t^4}, \frac{3}{t^4}\rangle,$$

and

$$\mathbf{r}'(t) = \langle 2t, 6t\rangle,$$

so

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = \int_1^2 \frac{1}{10^{3/2}} \left\langle \frac{1}{t^4}, \frac{3}{t^4} \right\rangle \cdot \langle 2t, 6t\rangle dt = \frac{1}{10^{3/2}} \int_1^2 \left(\frac{2}{t^3} + \frac{18}{t^3} \right) dt = \frac{3}{40}\sqrt{10}.$$