**Math 2153** - Spring 2017 Name: ______________________

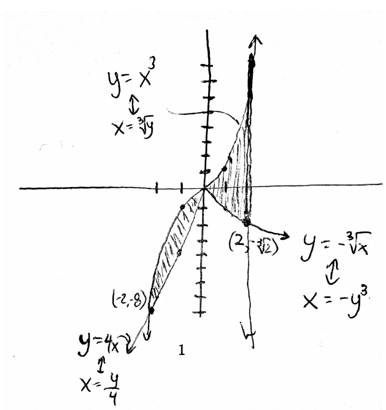
**Quiz 7** - Take Home (10 pts) Recitation Time: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, give me EXACT answers (do NOT use your decimals in your final answers, though they may be used to approximate where a number is in a graph).

**Problem 1** [1.5 pts] Switch the order of integration of $\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{0}\int_{0}^{\sqrt{25-x^2-y^2}} f(x,y,z)\,dzdydx$ to the order $dxdzdy$, changing the bounds of integration accordingly so that you're integrating the <u>same</u> region.

**Problem 2** [4 pts] Let $R$ represent the region in the plane in the $xy$-plane that is (1) bounded between $y = x^3$ and $y = 4x$ in the left half of the plane ($x \leq 0$); and (2) bounded between $y = x^3$, $y = -\sqrt[3]{x}$, and $x = 2$ in the right half of the plane $x \geq 0$).

(a). [1 pt] Provide a crude sketch of the above region, labeling the curves and points of intersection.




(b). [1 pt] Write the integral or sum of integrals that represent the area of $R$ with $y$ integrated first.

On $[-2,0]$, $x^3 \geq 4x$, and on $[0,2]$, $x^3 \geq -x^{1/3}$, so the area is $\displaystyle\int_{-2}^{0}\int_{4x}^{x^3} dydx + \int_{0}^{2}\int_{-x^{1/3}}^{x^3} dydx.$

(c). [1.5 pt] Write the integral or sum of integrals that represent the area of $R$ with $x$ integrated first.

On $[-8,0]$, $y^{1/3} \leq y/4$; on $[-2^{1/3},0]$, $-y^3 \leq 2$; on $[0{,}8]$, $y^{1/3} \leq 2$. Thus, the area is $\displaystyle\int_{-8}^{0}\int_{y^{1/3}}^{y/4} dxdy + \int_{0}^{8}\int_{y^{1/3}}^{2} dxdy + \int_{-2^{1/3}}^{0}\int_{-y^3}^{2} = dxdy.$

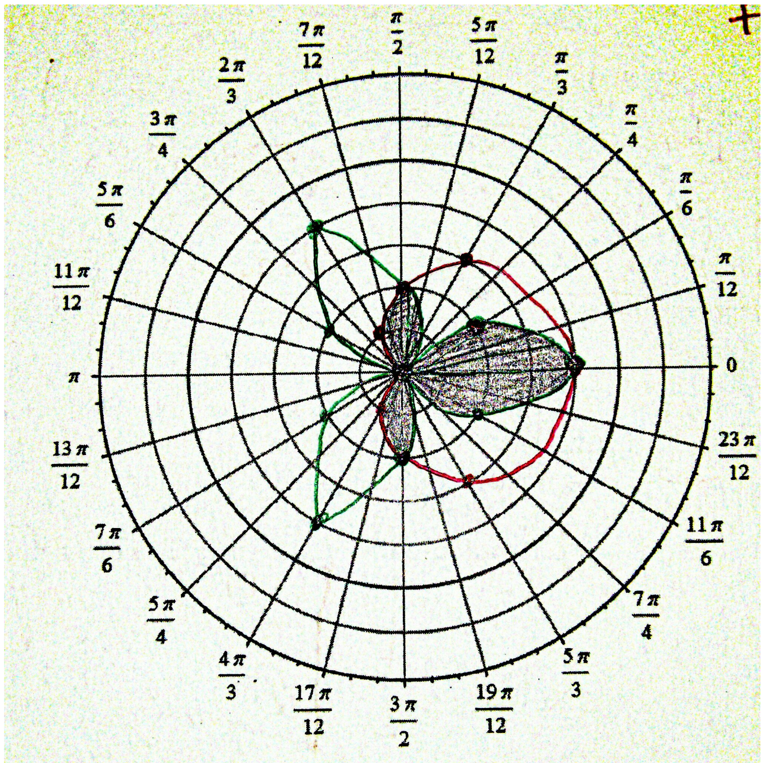
(d). [0.5 pts] Use either (b) or (c) to find the area of $R$.

Using (b), the area is

$\int_{-2}^{0} x^3 - 4xdx + \int_{0}^{2} x^3 + x^{1/3}dx = (\frac{x^4}{4} - 2x^2)\big|_{-2}^{0} + (\frac{x^4}{4} + \frac{3}{4}x^{4/3}\big|_{0}^{2} = 8 + \frac{3\cdot(2)^{1/3}}{2}.$

**Problem 3** [4.5 pts] Consider the region $R$ containing the origin bounded by the cardioid $r_1 = 2 + 2\cos(\theta)$ and the "three-petaled rose" $r_2 = 2 + 2\cos(3\theta)$, excluding the region between the curves.

(a). [2 pts] Graph the two curves by plotting 8 points $(r, \theta)$ for each curve (use multiple of $\pi/6$ that give you integer $r$ values), and shade in the region $R$ in the picture below. The shaded region should look like the Pokémon Diglett on its side. You need not include a table of values.




(b). [0.5 pts] Supposing you wanted to find the area between the curves instead, determine on which interval(s) $r_1$ is your "outer $r$" and on which interval(s) $r_2$ is your "outer $r$" for purposes of setting up an integral. Drawing rays in the picture above may be helpful.

$r_1$ is outer between $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$, and $r_2$ is outer for $\theta$ between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

(c). [2 pts] Set up and evaluate the integral or sum of integrals that represent the area of $R$ and evaluate it/them.

The area is

$$\int_{-\pi/2}^{\pi/2} \int_0^{2+2\cos(3\theta)} r dr d\theta + \int_{\pi/2}^{3\pi/2} \int_0^{2+2\cos(\theta)} r dr d\theta.$$

This is $\int_{-\pi/2}^{\pi/2} \frac{1}{2}(2 + 2\cos(3\theta))^2 d\theta + \int_{\pi/2}^{3\pi/2} \frac{1}{2}(2 + 2\cos(\theta))^2 d\theta$
$= 2\int_{-\pi/2}^{\pi/2} 1 + 2\cos(3\theta) + \cos^2(3\theta) d\theta + 2\int_{\pi/2}^{3\pi/2} 1 + 2\cos\theta + \cos^2\theta d\theta$. Applying the formula $\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$, we get the area is $6\pi - (8/3) - 8 = 6\pi - \frac{32}{3}$.