Math 2153 - Spring 2017 Name: ________________________

Quiz 5 - Take Home (10 pts) Recitation Time: ________________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, give me EXACT answers (do NOT use your decimals in your final answers, though they may be used to approximate where a number is in a graph).

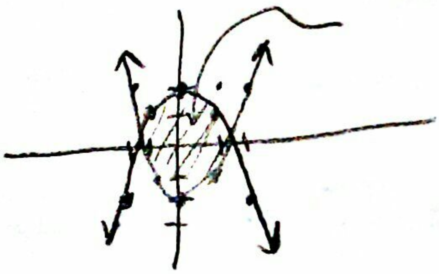
Problem 1 [10 pts] Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = \sqrt{y - x^2 + 2} + \sqrt{2 - x^2 - y}$.

(a). [0.5 pts] Find the domain of f. We'll call this set D in future parts of this problem.

$$y - x^2 + 2 \geq 0 \Rightarrow y \geq x^2 - 2$$

$$2 - x^2 - y \geq 0 \Rightarrow y \leq 2 - x^2$$

(b). [0.5 pts] Sketch D as a region in the xy-plane.



(c). [0.5 pts] Describe the interior and boundary points of the above region as *sets*. Do not just give examples. You may specially shade and point to these regions if you wish.

interior: region between parabolas

boundary: pts on parabolas

(d). [1 pt] State whether the conditions for the Strengthened Extreme Value Theorem (given in the handout) hold for f on D and how you know.

Since bdy pts included & region can be enclosed in finite disk, region is closed & bdd, so Extended EVT applies

(e). [1 pt] Find $\nabla f(x, y)$. You need not simplify completely.

$$\nabla f = \left\langle \frac{-x}{\sqrt{y-x^2+2}} - \frac{x}{\sqrt{2-x^2-y}}, \frac{1}{2\sqrt{y-x^2+2}} - \frac{1}{2\sqrt{2-x^2-y}} \right\rangle$$

(f). [1 pt] Find the equation of the tangent plane at $(0, 1, 1 + \sqrt{3})$.

$$\nabla f(0,1) = \left\langle 0, \frac{1}{2\sqrt{3}} - \frac{1}{2} \right\rangle$$

$$z = 1 + \sqrt{3} + \nabla f(0,1) \cdot \langle x - 0, y - 1 \rangle$$

$$\boxed{z = 1 + \sqrt{3} + \left(\frac{1}{2\sqrt{3}} - \frac{1}{2}\right)(y - 1)}$$

Problem 1 (ctd)
Recall that, as a consequence of the Extreme Value Theorem from first semester calculus, if $f : \mathbb{R} \to \mathbb{R}$ is a function that is differentiable on a closed interval $[a, b]$, then on $[a, b]$ f has an absolute maximum and absolute minimum, and these are attained either at the endpoints of $[a, b]$ or at points x in (a, b) where $f'(x) = 0$ or $f'(x)$ is undefined.

(g). [1 pt] Notice that f_x and f_y are both zero ONLY when $y = 0$ and that f_x and f_y are only undefined on the boundary of D. Using these facts and the extension of the Extreme Value Theorem from first semester calculus given above, find the critical point(s) of f in the *interior* of (meaning the complete collection of interior points in) the domain of f.

$g(x) = f(x,0) = 2\sqrt{2-x^2}$

$g'(x) = \frac{-2x}{\sqrt{2-x^2}} \Rightarrow$ crit pt at $(0,0)$ & $(\pm\sqrt{2}, 0)$

$f(0,0) = 2\sqrt{2}$, $f(\pm\sqrt{2}, 0) = 0$.

(h). [1 pt] Give separate parametrizations $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ for the two pieces of the boundary of the domain of f.

top half: $\vec{r}_1(t) = \langle t, -t^2+2 \rangle$ $-\sqrt{2} \le t \le \sqrt{2}$, bottom half: $\vec{r}_2(t) = \langle t, t^2-2 \rangle$ $-\sqrt{2} \le t \le \sqrt{2}$.

(j). [0.5 pts] Use part (h) to express f as a function g of t alone for each of the two pieces of the boundary (yes, you will have two different functions g_1 and g_2 for this).

$g_1(t) = f(r_1(t)) = \sqrt{4-2t^2}$

$g_2(t) = f(r_2(t)) = \sqrt{4-2t^2}$

(k). [1.5 pts] Use part (j) and the extension of the Extreme Value Theorem (1 variable) given at the top of the page to determine what the maximum and minimum values of f are on the boundary and where they occur. You will need both functions g_1 and g_2.

$g_1'(t) = g_2'(t) = \frac{1(-4t)}{2\sqrt{4-2t^2}} \Rightarrow$ CP's at $t = 0, \pm\sqrt{2}$, for both $\vec{r}_1$ & $\vec{r}_2$

$\vec{r}_1(0) = (0,2)$, $\vec{r}_1(\sqrt{2}) = (\sqrt{2}, 0)$, $\vec{r}_1(-\sqrt{2}) = (-\sqrt{2}, 0)$

$f(0,2) = 2$ $f(\pm\sqrt{2}, 0) = 0$

$\vec{r}_2(0) = (0,-2)$, $\vec{r}_2(\sqrt{2}) = (\sqrt{2}, 0)$, $\vec{r}_2(-\sqrt{2}) = (-\sqrt{2}, 0)$

$f(0,-2) = \sqrt{4} = 2$

max: 2 at $(0,-2)$ & $(0,2)$

min: 0 at $(\sqrt{2}, 0)$ & $(-\sqrt{2}, 0)$

(l). [1.5 pts] Use the previous parts to specify the absolute maximum and minimum values of f on its domain and where they occur (give points (x, y)). Cite the Strengthened Extreme Value Theorem (multivariable; given on recitation handout) if applicable.

Absolute max of f on D is $2\sqrt{2}$ at $(0,0)$

Absolute min of f on D is 0 at $(\pm\sqrt{2}, 0)$,

Since D is clsd & bdd & therefore Extended EVT says abs extrema of f on D exist & occur at crit pts in interior or on the bdy.