Math 2153 - Spring 2017 Name: KEY

Quiz 10 - Take Home (10 pts) Recitation Time: ______

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, please give me EXACT answers.

Problem 1 [2.5 pts] Let C be the octagon (oriented counterclockwise) obtained by connecting the following vertices in their given order with straight lines: (1,0), (2,0), (3,1), (3,2), (2,3), (1,3), (0,2), and (0,1); however, C has a missing edge - the one connecting (1,0) to (0,1). Consider the vector field $\mathbf{F} = \frac{\langle x,y\rangle}{(x^2+y^2)^{3/2}}$. Calculate $\int_C \mathbf{F}\cdot\mathbf{T}ds$. If you apply a theorem to simplify a computation, you must cite it and state why the conditions for the theorem apply.

$\vec{F} = \nabla\left(\frac{-1}{(x^2+y^2)^{1/2}}\right)$, so $\vec{F}$ is conservative.

Also, C is piecewise smooth (it's 7 line segments) & oriented counterclockwise starting at (1,0) & ending at (0,1). Therefore, by the FTC for line integrals,

$$\int_C \vec{F}\cdot\vec{T}ds = \left(\frac{-1}{(x^2+y^2)^{1/2}}\right)\Bigg|_{(x,y)=(1,0)}^{(x,y)=(0,1)} = \frac{-1}{\sqrt{1}} - \left(\frac{-1}{\sqrt{1}}\right) = 0.$$

Problem 2 [2.5 pts] Use a line integral to compute the area of the region $R = \{(x,y) : 9x^2+16y^2 \le 144\}$.

boundary is $9x^2+16y^2 = 144$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$.

Parametrize as $\vec{r}(t) = \langle 4\cos t, 3\sin t\rangle$, $0 \le t \le 2\pi$.

Area is $\oint_C x\,dy = \int_0^{2\pi} (4\cos t)(3\cos t)\,dt$

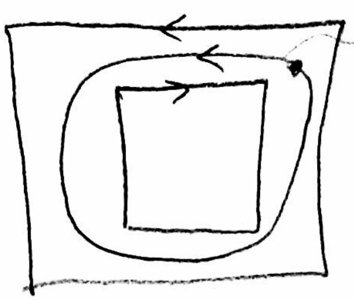
$= 12\int_0^{2\pi} \cos^2 t\,dt$

$= 12\int_0^{2\pi} \frac{1+\cos(2t)}{2}\,dt$

$= 12\left(\frac{t}{2} + \frac{\sin(2t)}{4}\right)\Big|_0^{2\pi} = \boxed{12\pi}$

Problem 3 [5 pts] Consider the region R bounded between the square with vertices at $(\pm1,\pm1)$ and the square with vertices at $(\pm2,\pm2)$. Suppose the outer part of the boundary is oriented counterclockwise and the inner part of the boundary is oriented clockwise. Also, consider the vector field $\mathbf{F} = \langle x^2y, xy^2\rangle$. We will calculate the flux of $\mathbf{F}$ on the boundary of R despite the fact Green's Theorem is not immediately applicable.

(a). [1 pt] Sketch R and a loop in R demonstrating that R is not simply connected.



can't contract this to a point because of hole in the middle

(b). [2 pts] Because R is not simply connected, Green's Theorem does not apply directly, but we can get around that. Describe a way to overcome this difficulty be decomposing R and its boundary as a combination or difference of two regions that Green's Theorem applies to, accounting for the orientation of the curves (Hint: Look at the margin notes in the examples in section 15.4; also, the fact that the inner boundary curve is oriented clockwise should provide you a clue).

We can overcome this by considering R as a big square minus a little square, both with boundary squares oriented counterclockwise. Indeed, subtracting the inner square reverses the orientation of its boundary, as desired, and both regions are simply connected.

(c). [2 pts] Compute the flux of $\mathbf{F}$ on $\cancel{C}$ the boundary of R, using Green's Theorem.

If C_1 is the outer part & C_2 is the inner part, both oriented counterclockwise

$$\text{flux} = \oint_{C_1} \vec{F}\cdot T\,ds - \oint_{C_2} \vec{F}\cdot\vec{T}\,ds$$

$$= \int_{-2}^{2}\int_{-2}^{2} 2xy+2xy\,dA - \int_{-1}^{1}\int_{-1}^{1} 2xy+2xy\,dA$$

$$= 2\left(\int_{-2}^{2} x\,dx\right)\left(\int_{-2}^{2} y\,dy\right) - 2\left(\int_{-1}^{1} x\,dx\right)\left(\int_{-1}^{1} y\,dy\right)$$

$$= 0 - 0 = 0.$$