Math 2153 - Spring 2017 Name: ______________________

Quiz 9 - Take Home (10 pts) Recitation Time: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, please give me EXACT answers.

Problem 1 [4 pts] Let D be the solid region bounded between the upper half of the ellipsoid $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} = 1$ and the plane $z = 0$.

(a). [0.5 pts] Find a transformation $T_1 : S \to D$ with $(u, v, w) \mapsto (x, y, z)$ where S is the upper half of the unit ball centered at the origin.

(b). [0.5 pts] As we learned in Section 14.5, S may be expressed in spherical coordinates via $u = \rho \sin\varphi \cos\theta$, $v = \rho \sin\varphi \sin\theta$, and $w = \rho \cos\varphi$. This describes a transformation $T_2 : R \to S$ from a region R in (ρ, φ, θ) 3D-space into S. Give a description R in (ρ, φ, θ) 3D-space.

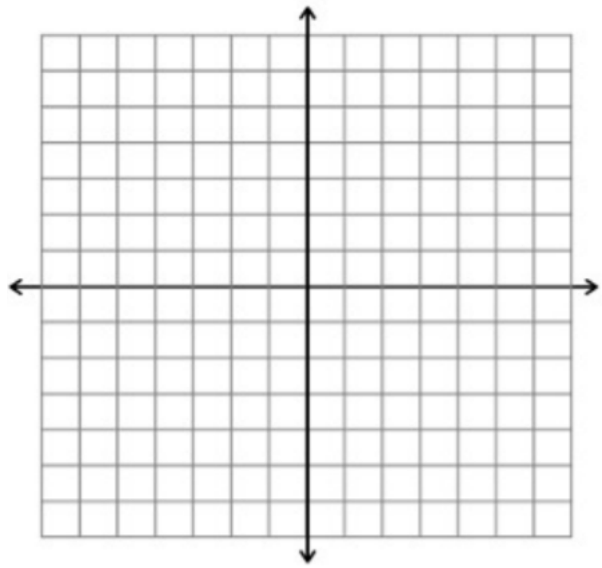
(c). [0.5 pts] Use the previous parts to express explicitly the transformation $T_1 \circ T_2 : R \to D$, $(\rho, \varphi, \theta) \mapsto (x, y, z)$.

(d). [1.5 pts] Find the Jacobian for the transformation $T_1 \circ T_2$. (Your final answer should be relatively simple after using the Pythagorean trig identity.)

(e.) [1 pt] Find $\iiint_D z\,dV$.

Problem 2 [5 pts] Consider the vector field $\mathbf{F} = \langle y - 2x, 2x - y \rangle$ and the curve C given by $\mathbf{r}(t) = \langle 2\cos t, 2\sin t \rangle$, $0 \leq t \leq 2\pi$.

(a). [2 pts] Sketch C (indicating orientation) and 12 vectors in the vector field for $\mathbf{F}$. Include at least 4 vectors with tails at points on C and try to space out your points (x, y) so that arrows $\mathbf{F}(x, y)$ don't land right on top of each other so much.



(b). [1 pt] Considering what vectors you plotted for your chosen points on C, make a prediction as to whether the flux will be positive negative, or zero, and explain your prediction using the intuitive meaning of flux.

(c). [2 pts] Compute the flux of $\mathbf{F}$ on C.

Problem 3 [1 pt] Suppose that $g : \mathbb{R}^2 \to \mathbb{R}$ is the potential function for a vector field $\mathbf{F}$.

(a). [0.5pts] State the simple but important fact about the vectors $\mathbf{F}(x, y)$ evaluated at points (x, y) on the level curves for g.

(b). [0.5pts] Consider one level curve C given by $g(x, y) = D$. Based on the intuitive definition of circulation, what do you think the circulation of $\mathbf{F}$ on C is?