Math 2153 - Spring 2017 Name: ______________________

Quiz 4 - Take Home (10 pts) Recitation Time: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, give me EXACT answers (do NOT use your decimals in your final answers, though they may be used to approximate where a number is in a graph).

Problem 1 [2 pt] For the function $h(u, v) = \left(\frac{uv}{u-v}\right)^{3/2}$, calculate the two first partial derivatives h_u and h_v.

Problem 2 [3 pts] Consider the function $G(x, y) = \sin^{-1}(y - x^2)$.

(a). [0.5 pts] Find the domain of G.

(b). [0.5 pts] Sketch the domain of G as a region in the xy-plane.

(c). [1 pt] State whether the above region is open, closed, or neither in the xy-plane and how you know. Include a description of what the interior and boundary points of the region are in your answer.

(d). [1 pt] State whether the region is bounded or unbounded in the xy-plane and how you know.

Problem 3 [5 pts] Consider the function $f(x,y) = \frac{\sin(xy)}{x+y}$.

(a). [1 pt] Evaluate what this limit would be along the path $y = mx$ for any m using the rule $\lim_{x\to 0} \dfrac{\sin(g(x))}{g(x)} = 1$ if $g(x) \to 0$ as $x \to 0$.

In the parts that follow, we will show that it is NOT enough to only consider the paths $y = mx$ to determine whether a limit exists, despite the fact they agree. Notice that as $x \to 0^+$, if we set $y = -\sin(x)$, then $y \to 0^-$ since $\sin(x) \to 0^+$ as $x \to 0^+$. So, this is yet another path to the origin as $x \to 0^+$. We'll show using Maclaurin series that the limit as (x,y) tends to $(0,0)$ along this path is $-\infty$.

(b). [1 pt] Find the Maclaurin series for $-x\sin(x)$ and $x - \sin(x)$.

(c). [1 pt] Find the first 3 terms for the Maclaurin series for $\sin(-x\sin x)$.

(d). [2 pts] Use parts (b) and (c) to show that as $x \to 0^+$ along the path $y = -\sin(x)$ $f(x,y)$ tends to $-\infty$. You are encouraged to just write the first 3 nonzero terms of each Maclaurin series and "$\cdots$" for subsequent terms (please DON'T write the full Σ expression). Using methods like in Example 3 on pages 91-93 in the textbook should be helpful in evaluating the resulting limit.

Note: Notice how much easier the method in part (d) is than using L'Hospital's Rule. That requires 3 separate applications and UGLY Chain and Product Rule computations! You can generally get away with using this method instead of L'Hospital's Rule for evaluating a variety of limits.