Math 2153 - Spring 2017 Name: ______________________

Quiz 10 - Take Home (10 pts) Recitation Time: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Furthermore, please give me EXACT answers.

Problem 1 [2.5 pts] Let C be the octagon (oriented counterclockwise) obtained by connecting the following vertices in their given order with straight lines: (1,0), (2,0), (3,1), (3,2), (2,3), (1,3), (0,2), and (0,1); however, C has a missing edge - the one connecting (1,0) to (0,1). Consider the vector field $\mathbf{F} = \frac{\langle x,y \rangle}{(x^2+y^2)^{3/2}}$. Calculate $\int_C \mathbf{F} \cdot \mathbf{T} ds$. If you apply a theorem to simplify a computation, you must cite it and state why the conditions for the theorem apply.

Problem 2 [2.5 pts] Use a line integral to compute the area of the region $R = \{(x, y) : 9x^2 + 16y^2 \leq 144\}$.

Problem 3 [5 pts] Consider the region R bounded between the square with vertices at $(\pm 1, \pm 1)$ and the square with vertices at $(\pm 2, \pm 2)$. The boundary of R has two pieces. Suppose the outer part of the boundary is oriented counterclockwise and the inner part of the boundary is oriented clockwise. Also, consider the vector field $\mathbf{F} = \langle x^2 y, xy^2 \rangle$. We will calculate the flux of $\mathbf{F}$ on the boundary of R despite the fact Green's Theorem is not immediately applicable.

(a). [1 pt] Sketch R and a loop in R demonstrating that R is not simply connected.

(b). [2 pts] Because R is not simply connected, Green's Theorem does not apply directly, but we can get around that. Describe a way to overcome this difficulty be decomposing R and its boundary as a combination or difference of two regions that Green's Theorem applies to, accounting for the orientation of the curves. (Hint: Look at the margin notes in the examples in section 15.4; also, the fact that the inner boundary curve is oriented clockwise should provide you a clue. What does reversing the orientation of a curve in a line integral do to the line integral?).

(c). [2 pts] Using the idea from part (b), compute the flux of $\mathbf{F}$ on the boundary of R (which has two pieces), using Green's Theorem. You need not write the line integral(s).