

① Use elementary Row Operations to RREF

$$A = \left[\begin{array}{cccc|ccccc} 0 & 0 & 0 & 0 & 2 & 8 & 4 \\ 0 & 0 & 0 & 1 & 3 & 11 & 9 \\ 0 & 3 & -12 & -3 & -9 & -24 & -33 \\ 0 & -2 & 8 & 1 & 6 & 17 & 21 \end{array} \right]$$

② Solve the following system.

$$x_1 + x_2 - x_5 = 1$$

$$x_2 + 2x_3 + x_4 + 3x_5 = 1$$

$$x_1 - x_3 + x_4 + x_5 = 0$$

Describe solution in parametric vector form
and give geometric description.

③ $x_1 + 3x_2 - x_3 = b_1$

$$x_1 + 2x_2 = b_2$$

$$3x_1 + 7x_2 - x_3 = b_3$$

Determine conditions on b_1, b_2, b_3 that are necessary and sufficient for the system to be consistent.

④ Are vectors $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$ linearly dependent?

⑤ $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$

Are vectors $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ linearly dependent?

Yes, they are

linearly dependent, because $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$, i.e., $1 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (-1) \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$

$$\textcircled{6} \quad C = \begin{bmatrix} 2 & 1 \\ 4 & 0 \\ 8 & -1 \\ 3 & 2 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 2 & 0 & 0 & 4 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute $A + B$.

Compute DC ? Is CD well defined?

Compute C^T . Is $C^T D$ well defined?

\textcircled{7} The given matrix is the augmented matrix for a system of linear equations.

Give the vector form for the general solution.

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array} \right]$$

\textcircled{8} Determine conditions on the scalars so that the set of vectors is linearly dependent.

a) $\tilde{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \tilde{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \tilde{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix}$

b) $\tilde{v}_1 = \begin{bmatrix} a \\ 1 \\ 1 \end{bmatrix}, \tilde{v}_2 = \begin{bmatrix} b \\ 3 \\ 1 \end{bmatrix}$

(9) Are matrices $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$\begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix}$ singular or non-singular?

(10) Find constants so that the given function satisfies the given conditions

$$y = c_1 e^{-x} + c_2 e^x + c_3 e^{2x}$$

$$y(0) = 8, \quad y'(0) = 3, \quad y''(0) = 11.$$

(11) When $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a solution of the given system?

Solution:

$$\left[\begin{array}{cc|c} a & b & 1 \\ b & a & -1 \end{array} \right] \text{ is the same as } \begin{cases} ax_1 + bx_2 = 1 \\ bx_1 + ax_2 = -1 \end{cases}$$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a solution of the given system only if

the following equalities are true $a + 2b = 1$
 $b + 2a = -1$

Therefore, to guarantee that $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a solution of $\left[\begin{array}{cc|c} a & b & 1 \\ b & a & -1 \end{array} \right]$
we need to find a, b such that $a + 2b = 1$
and $b + 2a = -1$.

$$\begin{cases} a + 2b = 1 \\ b + 2a = -1 \end{cases} \rightarrow \begin{cases} a + 2b = 1 \\ 2a + b = -1 \end{cases} \xrightarrow{\text{solving system}} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 1 & -1 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -3 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array} \right] \rightarrow \boxed{a = -1 \text{ and } b = 1}$$