Homework 5

Math 2177, Lecturer: Alena Erchenko

1. Evaluate the integral
$$\iiint_B z\, dV$$
where B is the unit ball centered at the origin.

2. Set up an integral for the volume of the region bounded by $z = \sqrt{3(x^2+y^2)}$ and $x^2+y^2+z^2 = 4$ in all three coordinate systems (Euclidean, cylindrical, spherical). Evaluate whichever you prefer.

3. Sketch the vector fields
$$\vec{F}(x,y) = \frac{y\hat{\imath} - x\hat{\jmath}}{\sqrt{x^2+y^2}} \qquad \text{and} \qquad \vec{G}(x,y,z) = \hat{\imath} + \hat{k}.$$

4. Compute the line integral
$$\int_C x\, ds$$
where C is the line segment from $(1,0)$ to $(0,1)$. Do this for two different parameterizations of the line segment and check that you get the same answer.

5. Compute $\int_C \vec{F} \cdot d\vec{r}$ where C is the closed curve obtained from traveling along the x-axis from the origin to $(1,0)$ then along the unit circle to the point $(1/\sqrt{2}, 1/\sqrt{2})$ and finally back to the origin along a straight line and $\vec{F} = y\hat{\imath} + 2x\hat{\jmath}$.

6. Compute directly $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \langle y^2 + 2xy, x^2 + 2xy \rangle$ and C is the portion of the cusp curve $y^2 = x^3$ from the origin to the point $(1,1)$. Use the theorem in class to check that $\vec{F}$ is conservative. Recall that the work of a conservative field along a closed curve is equal to 0. Use this to replace C by a simpler curve, and compute the same integral using this curve. Finally, find a potential function for f and compute the integral using the fundamental theorem for line integrals.

7. Compute $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \langle (1+xy)e^{xy}, x^2 e^{xy} \rangle$ and C is the curve with parameterization $\vec{r}(t) = \langle t^4 + 7t, t\cos(\pi t) \rangle$ for $0 \le t \le 1$. (Hint: is $\vec{F}$ conservative?)