

Lab 8: Filters in MATLAB

ReadMeFirst

Lab Summary

In this final lab you will bridge the gap between the analog world and the world of digital filters by beginning with a real (passive) filter and using the bilinear transform to make an equivalent filter in the Z domain. In the process you should gain a better understanding of some of the more useful functions in MATLAB that should help you in future courses. There are subtleties and idiosyncrasies in MATLAB that this lab should give you the tools to work around.

Lab Preparation

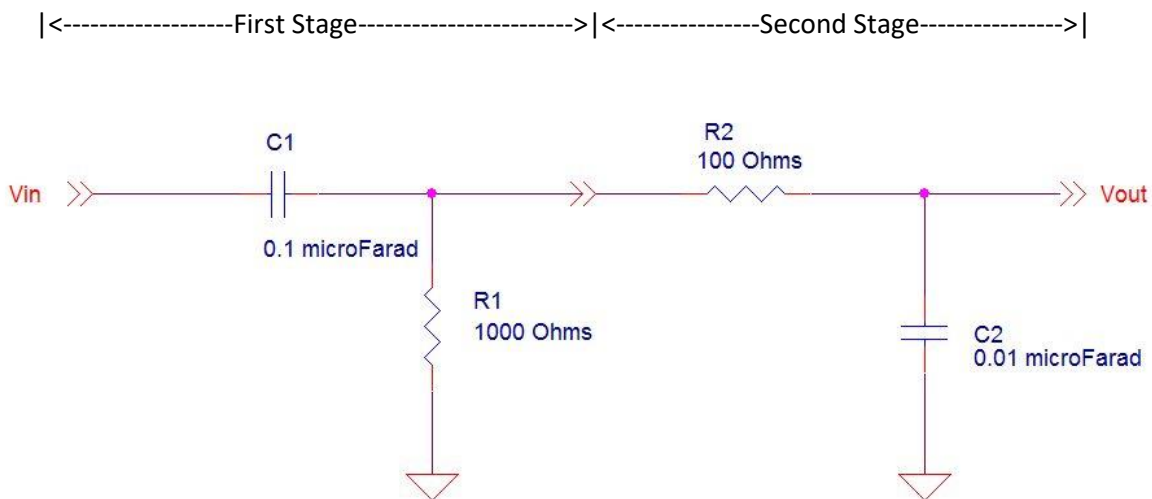
NONE

Lab Supplies

NONE

Part 1 - The Analog Filter

Consider the following analog circuit:

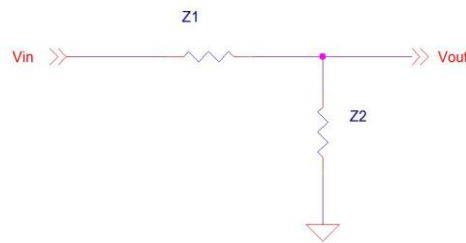


Using Laplace notation derive the transfer function for this passive filter $H(s)$. You can derive the low pass portion and the high pass portion independently and multiply them together. If you use R and $1/sC$ for impedances and apply the voltage divider equation to each stage and then multiply them together, you should arrive at the correct expression.

NOTE: If you are unfamiliar with LaPlace, just think of s as a substitution for $j\omega$. So for a capacitor the impedance would be:

$$\frac{1}{j\omega C} = \frac{1}{sC}$$

The voltage divider equation is:



$$V_{out} = V_{in} * \frac{Z_2}{Z_1 + Z_2}$$

and your transfer function is :

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

Now use the coefficients in MATLAB to plot the frequency response of this analog filter. Use combinations of R_1 , R_2 , C_1 , and C_2 for your coefficients **b** (numerator) and **a** (denominator).

Write out your answer for the $H(s)$.

Check Point 1: Ask a Lab Monitor or TA to verify your $H(s)$ before proceeding.

Generate a logarithmic time base for your BODE plot and use the **bode** command for your plot. There is a form that uses b, a, and w, where w is the range of the bode plot. For w, use

```
w = logspace(3, 7, 2048); % This generates a logarithmic range of 2048 points from 1000 to 10,000,000.
```

Recall that the cut-off frequencies are $1/RC$ in radians per second for both high pass and low pass filters. Also recall that the cut-off frequencies occur at -3 decibels in the bode plot.

What kind of filter is this? What are your calculated cut-off frequencies? What cutoff frequencies are shown on the plot? What are the phase shifts at the cutoff frequencies?

NOTE: You must include these answers in your Lab report.

Part 2 - The Bilinear Transform

In this part you will use the Bilinear Transform in order to convert your analog filter to an equivalent digital filter in the Z domain. This will be an Infinite Impulse Response filter.

The Bilinear Transform converts an S-domain transfer function to a Z-domain transfer function $H(z)$ by making the following substitution:

$$s = 2 * F_s * \frac{z - 1}{z + 1}$$

where F_s is the sampling frequency for the new digital filter. Set F_s to 2^{21} which is equal to 2,097,172.

You could perform the bilinear transformation manually, but MATLAB has a bilinear function that does this for you. You can generate the numerator coefficients and denominator coefficients for $H(z)$ from b, a , and F_s as follows:

```
[NumZ, DenZ] = bilinear(b, a, Fs)
```

Using the coefficients, write out $H(z)$.

Check Point 2: Ask a Lab Monitor or TA to verify your $H(z)$ before proceeding.

Part 3 - The Digital Filter in the Z-plane

To help you understand where zeros and poles end up in the Z-plane and how the plot changes with cut-off frequencies, there are two "movies" that have been generated in MATLAB, one for Low pass filters and one for High pass filters. Download the M-files *IIR_Low.m* and *IIR_High.m* from the ECE 2050 website and run them in MATLAB.

Now plot your IIR filter on the Z-plane using

```
zplane(NumZ, DenZ);
```

NOTE: In the Discussion Questions, you are asked to sort out which poles and zeros belong to which stage of the analog filter using the transfer functions for each of filter stage separately, the bilinear transform, and the zplane plot. Now is a good time to generate those plots. Start by calculating $H(s)$ for each stage separately. Then perform the bilinear transform on each and plot separate z-plane images.

Check Point 3: Ask a Lab Monitor or TA to verify your Z-plane plot.

Part 4 - The Digital IIR Filter Response

In the final part you will plot the frequency response of your digital filter and compare it to the analog response. Begin by plotting your filter response using:

```
freqz(Num, Den, 10240); % The last number just tells MATLAB how many points to plot.
```

Notice that the plot is Normalized Frequency. OK that is REALLY annoying, but the people at Mathworks LOVE IT! It is completely and utterly useless. The frequencies are expressed as are the portion of the sampling frequency that corresponds to each REAL frequency in radians per second.

Here is how to convert the answer to something that you can compare to your original analog filter. Generate the following frequency response **H** and frequency base **ww** using the following command

```
[H,ww] = freqz(NumZ, DenZ, 10240);
```

Convert the frequency base from Normalized Frequency to radians per second by multiplying by the sampling frequency Fs:

```
w2 = ww*Fs; % Convert from normalize freq to radians per second
```

For a plot equivalent to the BODE plot, you will need to convert the magnitude to decibels. You must first take the absolute value of H and then convert to decibels. Decibels is a logarithmic scale and the definition is

$$\text{decibels} = 20 * \log_{10}(\text{Gain})$$

so the two following lines are equivalent:

```
y = 20*log10(abs(H)); % Convert to Decibels  
y = db(abs(H)); % Convert to Decibels
```

To plot more than one graph at a time, you can use the subplot command.

```
subplot(2,1,1);
```

The first number is the number of rows. The second number is the number of columns, the third number is which subplot you are selecting. So if you wanted to plot one graph on top of the other, you would begin by executing subplot(2,1,1) followed by any plot command. The second plot is accessed by subplot(2,1,2).

The plot command that is used to generate a BODE plot MANUALLY is **semilogx**. This makes the x axis logarithmic

Now generate the Magnitude Plot using w2 and y. Select the upper subplot and execute the following command.

```
semilogx(w2, y);
```

To generate the Phase plot, first select the lower subplot. You can still use semilogx and w2 as the x axis. The Phase Angle take some doing. Begin by extracting the angle from H:

```
angle(H)
```

This is in radians however, and should be converted to degrees. Fortunately MATLAB has a **rad2deg** function. So the entire line for the Phase Angle subplot would look like this:

```
semilogx(w2, (rad2deg(angle(H))))
```

Again there are a number of changes that you can make to make your result more readable. You MUST turn on the grid in a logarithmic plot in order to see what is really being plotted:

```
grid on;
```

You can match the limits to the Analog Bode plot that you did earlier by declaring X and Y axis limits:

```
axis([1000 10000000 -20 0]);
```

You will need to identify the cut-off frequencies so it would be nice if -3 dB was shown on the graph. You can force the y axis to begin at -21 and increment to 0 in steps of 3 as follows:

```
set(gca, 'YTick', [-21:3:0])
```

You will usually never get credit for graphs unless your X and Y axes are labeled. It is also an excellent idea to add a title

```
title('Magnitude BODE Plot of IIR Bandpass Filter');  
xlabel('Frequency log10(rads/sec)');  
ylabel('Gain (dB)');
```

Use the axis command to match the X range to the Magnitude plot, but force the Y axis to go from -90 to 90. It is also useful to see the Phase angle at +/- 45 degrees and 0 degrees. Use the set command to make the YTicks go from -90 to 90 in increments of 45 degrees. Label the X and Y axis appropriately.

```
axis([1000 10000000 -90 90]);  
grid on;  
set(gca, 'YTick', [-90:45:90])  
xlabel('Frequency log10(radians/second)');  
ylabel('Phase Angle (degrees)');
```

Check Point 4: Ask a Lab Monitor or TA to verify your IIR Bode plot.

Now examine the Bode plot of the IIR filter. What are the cutoff frequencies? What is the phase angle at each cut-off frequency? What is the phase angle in the exact middle of the passband? What frequency corresponds to the High Pass Stage?

Once you have generated your code, copy the last part where you manipulated the freqz to look like a bode plot. Now suppose you want the plot to be in Hertz instead of Radians per second. How would you do it? Recall that

$$\omega(\text{rads/sec}) = 2 * \pi * f(\text{Hz})$$

so if you begin with Normalized Frequency w_w , multiply by the sampling frequency and divide by $(2*\pi)$ to convert the axis variable to Hertz:

```
w2 = ww*Fs/(2*pi);           % Convert from normalized freq to Hertz
```

Re-plot the IIR frequency response in Hertz instead of radians/second. Change your labels accordingly.

Check Point 5: Ask a Lab Monitor or TA to verify your IIR Bode plot in Hertz.

Part 5 - Comparison of the Filters

Copy the following code into MATLAB and run it. If you used different variable names throughout your code, make the necessary changes to get the plot to work. Use the plot to comment on the difference between the Magnitude responses of the original analog filter and the digital IIR filter that was designed using the Bilinear Transform. Convert your digital IIR frequency (w_2) back to radians per second if it is still in Hertz.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Comparison of Analog and Digital IIR Filter%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

close all;
[mag,phase] = bode(b,a,w);
semilogx(w,db(abs(mag)), 'b',w2,y, 'r');
grid on;
axis([1000 10000000 -20 0]);
legend('analog','digital IIR');
title('Comparison of Magnitude Plots for Analog and IIR Bandpass Filter');
xlabel('Frequency log10(rads/sec)');
ylabel('Gain (dB)');
```

NOTE: Please include all transfer functions and all plots in your lab report.

Check Point 6: Ask a Lab Monitor or TA to verify your Lab Bench is cleaned and prepared for the next lab group.

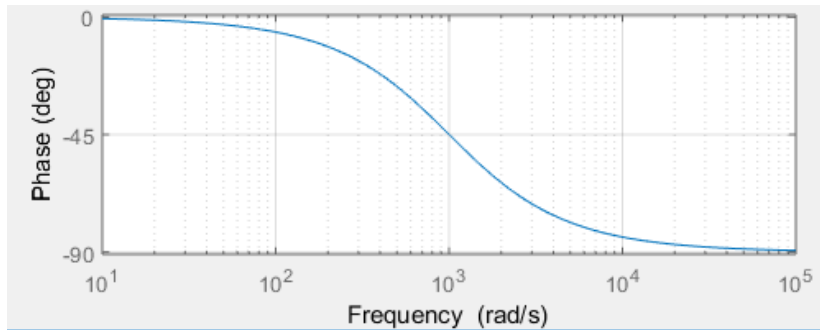
Discussion Questions:

Topic 1: Using separate transfer functions for the Low pass stage and the High Pass Stage of the filter in this lab, plot the poles and zeros on the Z plane to sort out which belong to which filter in the Part 3 of the Lab.

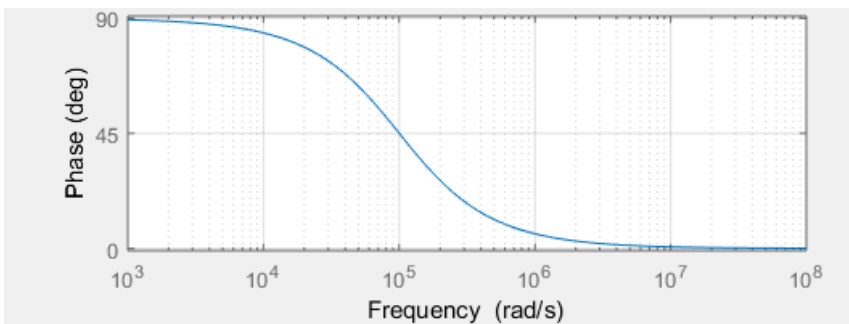
Topic 2: In your filter, what frequency (rads/sec) corresponds to the cut-off frequency of the Low Pass stage? Is this Stage 1 or 2 on the original diagram?

Topic 3: In your filter, what frequency (rad/sec) corresponds to the cut-off frequency of the High Pass stage? Is this Stage 1 or 2 on the original diagram?

Topic 4: Here is a Phase angle plot for a Low Pass filter. What is the cutoff frequency in radians/second?



Topic 5: Here is a Phase angle Plot for a High Pass Filter. What is the cutoff frequency?



Topic 6: Briefly explain Normalized Frequency and how to convert it to radians per second, and how to convert it to Hertz.

Topic 7: What are the two plots in a Bode plot? What are the X and Y axis units for each of the two plots?

*** END of LABS ***