ICP: Solution algorithms

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What

Barrier

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Only inequality constraints!

We use a generic inequality-constrained nonlinear optimization problem of the form:

Barrier

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & f(x) \\ \text{s.t.} \quad & g_1(x) \leq 0 \\ & g_2(x) \leq 0 \\ & \vdots \\ & g_r(x) \leq 0 \end{aligned}$$

Only ≤ inequality constraints!

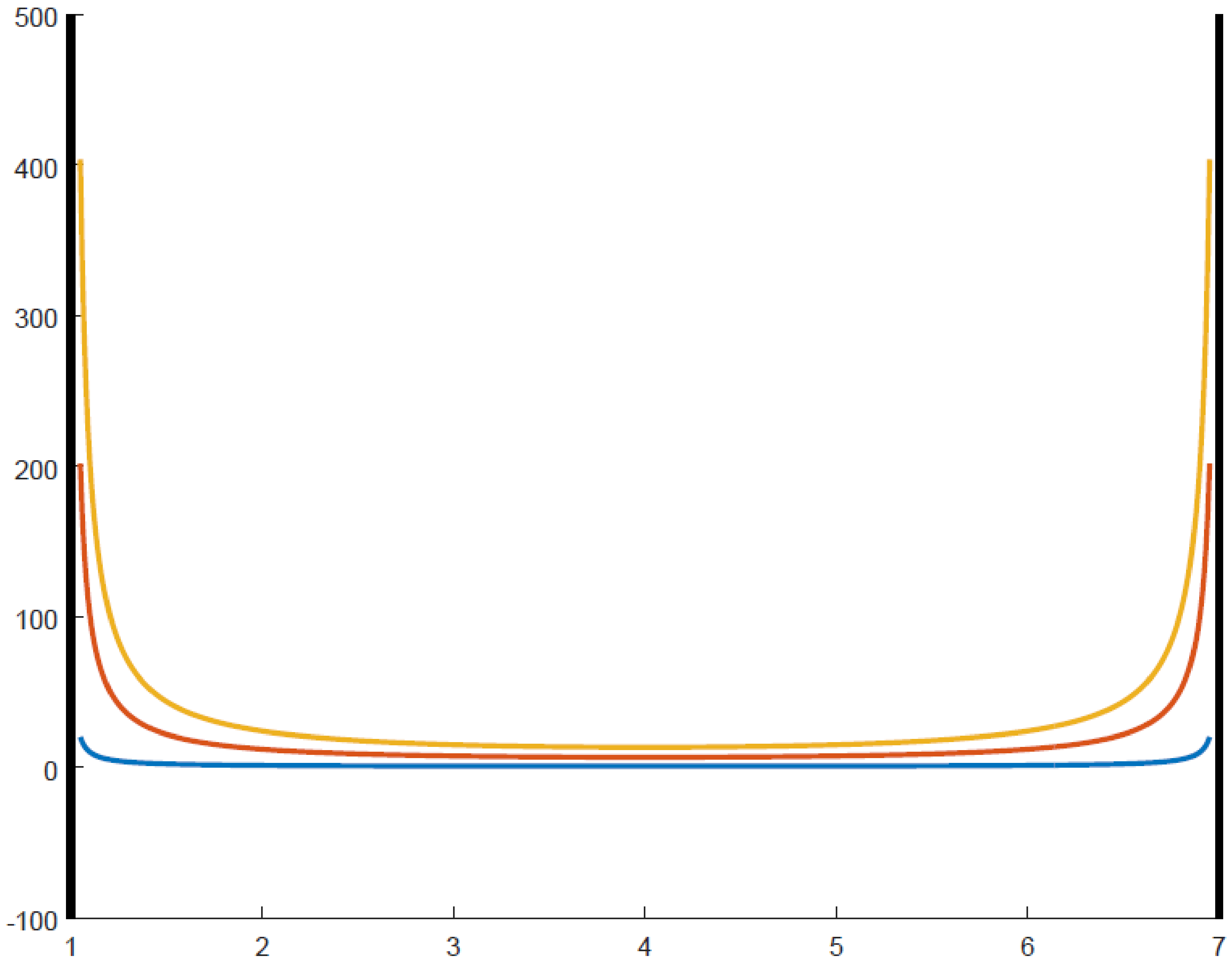
Barrier

Barrier as we approach 0

$$f(x) - \sum_{j=1}^{r} \frac{1}{\phi_j} \cdot \frac{1}{g_j(x)}$$

where $\phi_1, \phi_2, \ldots, \phi_r > 0$ are fixed coefficients that determine how much weight is placed on violating a particular constraint.

Barrier



Barrier

We solve this!

$$\min_{x \in \mathbb{R}^n} \mathcal{F}_\phi(x) = f(x) - \sum_{j=1}^{r} \frac{1}{\phi_j} \cdot \frac{1}{g_j(x)}$$

Barrier

In practice, we begin with small values for the ϕ's (setting them equal to one is a reasonable starting point) and begin using an iterative algorithm for UPs.

As we conduct iterations, we test to see if the constraint violations are getting smaller or not.

Barrier

The ϕ's associated with constraints that are having their violations reduced are kept the same. The ϕ's associated with constraints that are not seeing an improvement in their violations are increased.

This is continued iteratively until all of the constraints are satisfied and the algorithm finds a stationary point of $\mathcal{F}(x)$, or another termination criteria (such as a limit on the number of iterations) is met.

Barrier

1. Set $k = 0$ and initialize x^k and ϕ^k.
2. Solve: $\min_{x \in \mathbb{R}^n} \ \mathcal{F}_{\phi^k}(x)$, and get x^{k+1}.
3. If

   $|x^{k+1} - x^k| \leq \epsilon_1$ and

   $g(x^{k+1}) \leq \epsilon_2$, stop.
4. Otherwise update ϕ as appropriate and continue in 2.

Barrier: Solve: $\min_{x \in \mathbb{R}^n} \; \mathcal{F}_{\phi^k}(x)$, and get x^{k+1}.

procedure GENERIC ITERATIVE ALGORITHM
    $k \leftarrow 0$ ▷ Set iteration counter to 0
    Fix x^0 ▷ Fix a starting point
    while Termination criteria are not met **do**
        Find direction, d^k, to move in
        Determine step size, α^k
        $x^{k+1} \leftarrow x^k + \alpha^k d^k$ ▷ Update point
        $k \leftarrow k + 1$ ▷ Update iteration counter
    end while
end procedure

Barrier: Example

Barrier: Example

Consider the problem:

$$\begin{aligned} \min_{x} \ & f(x) \\ \text{s.t. } & a - x \leq 0 \\ & x - b \leq 0. \end{aligned}$$

Barrier: Example

$$\min_{x} \; 5(x-10)^2 - 20$$
$$\text{s.t. } 1 - x \leq 0$$
$$x - 7 \leq 0$$

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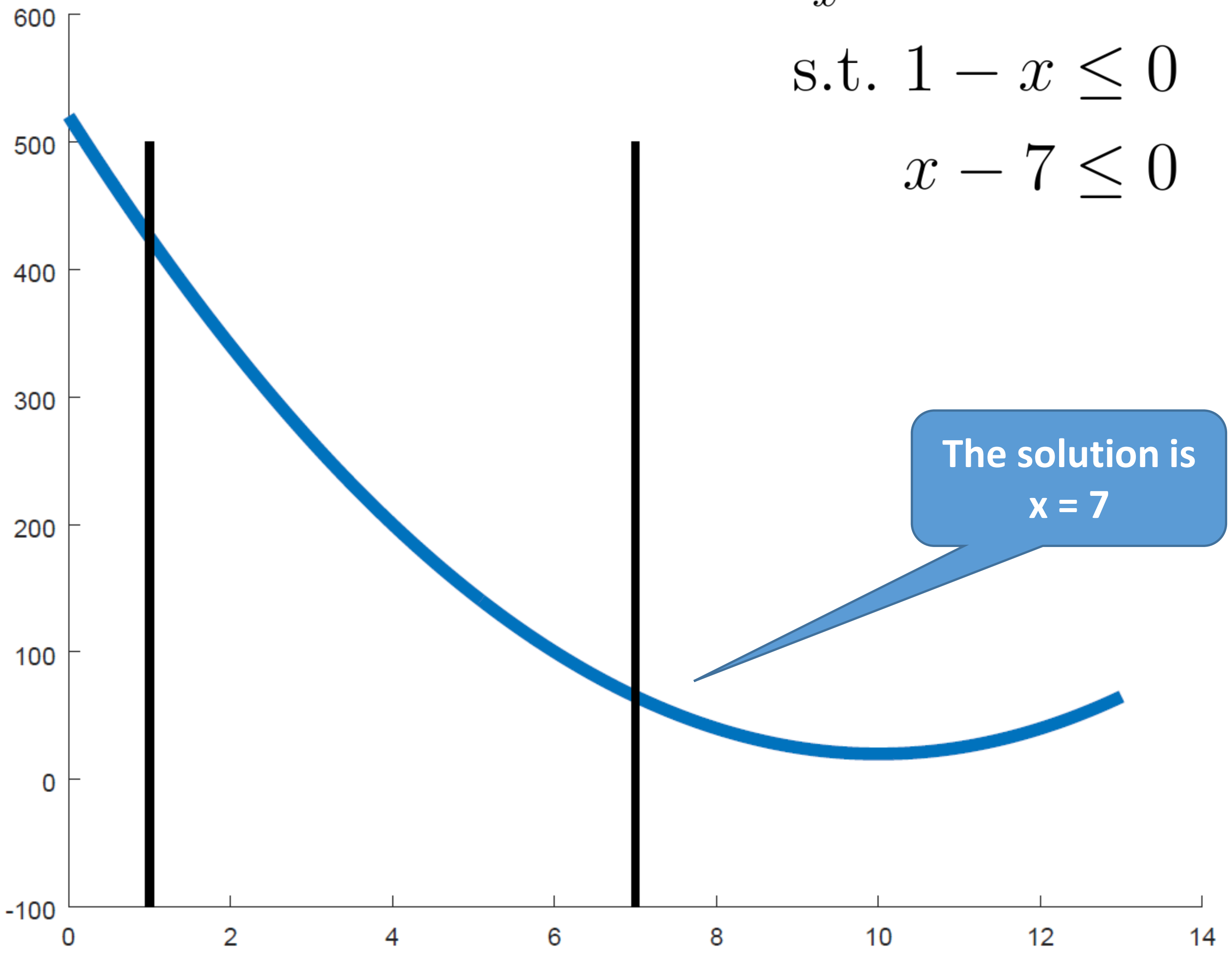
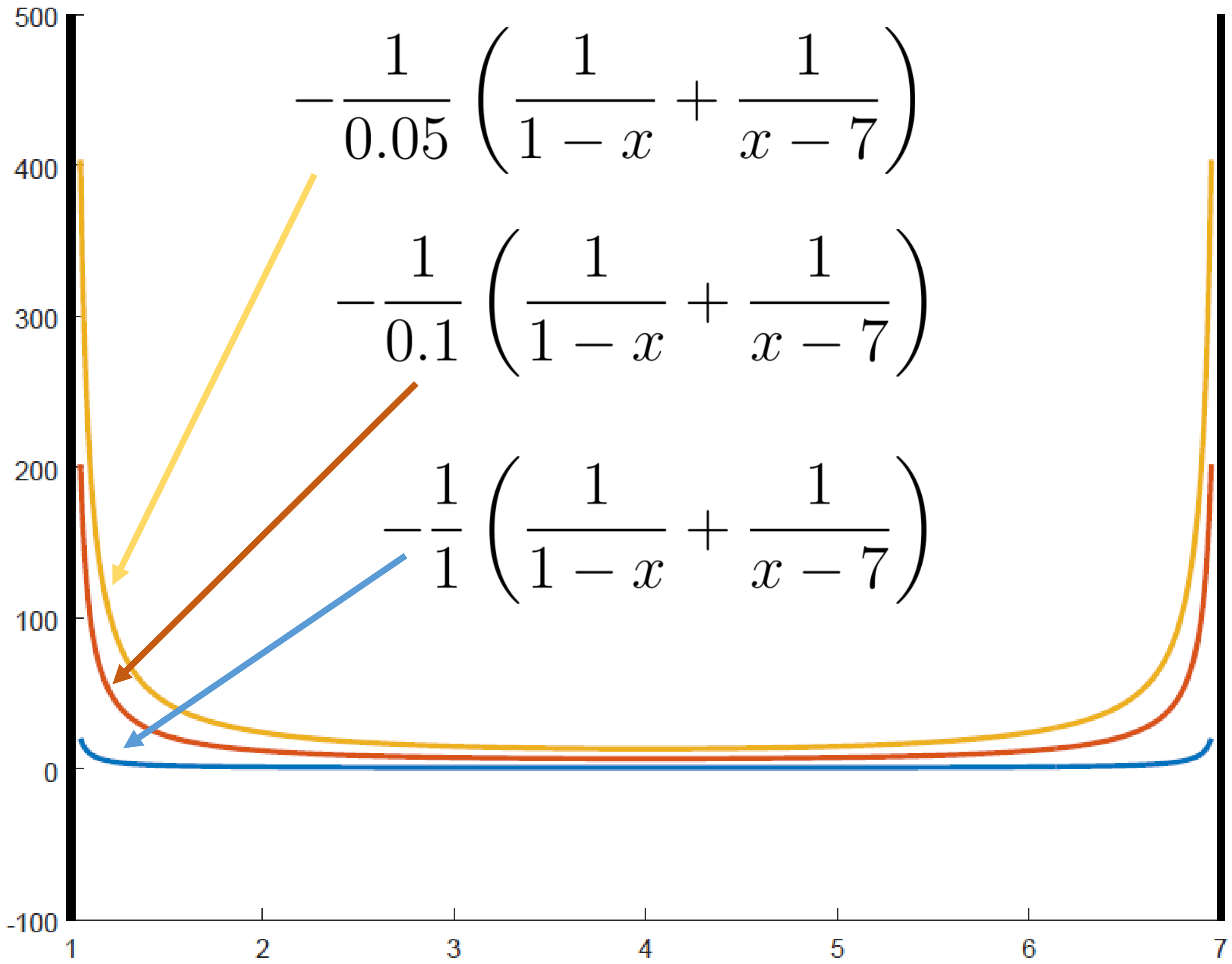
$$\min_x \; 5(x-10)^2 - 20$$
$$\text{s.t. } 1 - x \leq 0$$
$$x - 7 \leq 0$$



Barrier: Example

```
clear
x = 0:1/100:13;
y = 5*(x-10).^2+20;
hold
plot(x,y,'LineWidth',5)
line([1 1],[-100 500],'LineWidth',3);
line([7 7],[-100 500],'LineWidth',3);
```

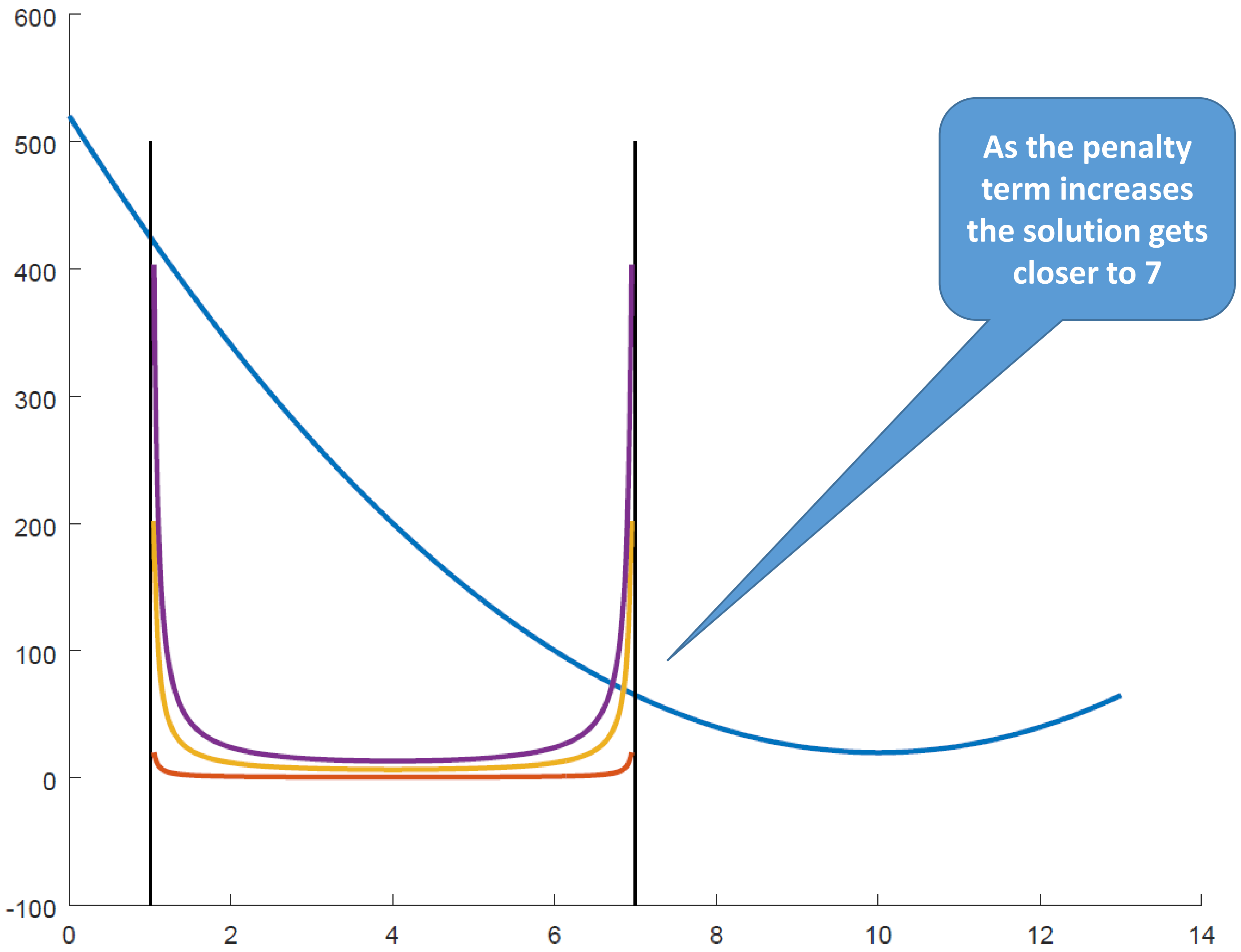
Barrier: Example



Barrier: Example

```
line([1 1],[-100 500],'LineWidth',3);
line([7 7],[-100 500],'LineWidth',3);
hold on
x= 1.05:1/100:6.95;
y1=-(1/1)*((1./(1-x))+(1./(x-7)));
plot(x,y1,'LineWidth',2)
y2=-(1/0.1)*((1./(1-x))+(1./(x-7)));
plot(x,y2,'LineWidth',2)
y3=-(1/0.05)*((1./(1-x))+(1./(x-7)));
plot(x,y3,'LineWidth',2)
```

Barrier: Example



Barrier: Example

```
clear
x = 0:1/100:13;
y = 5*(x-10).^2+20;
hold
plot(x,y,'LineWidth',2)
line([1 1],[-100 500],'LineWidth',1);
line([7 7],[-100 500],'LineWidth',1);
hold on
x= 1.05:1/100:6.95;
y1=-(1/1)*((1./(1-x))+(1./(x-7)));
plot(x,y1,'LineWidth',2)
y2=-(1/0.1)*((1./(1-x))+(1./(x-7)));
plot(x,y2,'LineWidth',2)
y3=-(1/0.05)*((1./(1-x))+(1./(x-7)));
plot(x,y3,'LineWidth',2)
```

GAMS

GAMS

$$\min_{x} \; 5(x-10)^2 - 20$$

$$\text{s.t.} \; 1 - x \leq 0$$

$$x - 7 \leq 0$$

```
variable x, z;
equations fo, wall1, wall2;
fo.. z =e= 5*power((x-10),2)-20;
wall1..  1-x =l= 0;
wall2..  x-7 =l= 0;
model barrier /all/;
solve barrier using nlp minimizing z;
option decimals = 8;
display x.l, z.l;
```

GAMS

$$\min_{x} \ 5(x-10)^2 - 20$$

$$\text{s.t. } 1 - x \leq 0$$

$$x - 7 \leq 0$$

```
----      9 VARIABLE x.L                    =    7.00000000
            VARIABLE z.L                    =   25.00000000
```

A Different Barrier

$$\begin{aligned} \min_{x} \;& 5(x-10)^2 - 20 \\ \text{s.t. } & 1 - x \leq 0 \\ & x - 7 \leq 0 \end{aligned}$$

$$\min_{x \in \mathbb{R}^n} \; \mathcal{F}_\phi(x) = f(x) - \sum_{j=1}^{r} \frac{1}{\phi_j} \cdot \log(-g_j(x))$$

A different barrier!

A Different Barrier

$$\min_{x} \; 5(x-10)^2 - 20$$
$$\text{s.t. } 1 - x \leq 0$$
$$x - 7 \leq 0$$

$$\min_{x} \; [5(x-10)^2 - 20] - \left[\frac{1}{\phi}\log(-(1-x)) + \frac{1}{\phi}\log(-(x-7))\right]$$

GAMS

$$\min_x \; \left[5(x-10)^2 - 20\right] - \left[\frac{1}{\phi}\log(-(1-x)) + \frac{1}{\phi}\log(-(x-7))\right]$$

```
scalar phi /10000000/;
variables x, z;
x.l=4;
equation fo;
fo.. z =e= 5*power((x-10),2)+20 - (1/phi)*( log(-(1-x))+log(-(x-7)) );
model barrier /all/;
solve barrier using nlp minimizing z;
option decimals = 8;
display phi, x.l, z.l;
```

GAMS

$$\min_{x} \; \left[5(x-10)^2 - 20\right] - \left[\frac{1}{\phi}\log(-(1-x)) + \frac{1}{\phi}\log(-(x-7))\right]$$

```
----       9 PARAMETER phi                    =   1.00000000
             VARIABLE x.L                     =   6.96720828
             VARIABLE z.L                     =  67.62042813

----       9 PARAMETER phi                    =  10.00000000
             VARIABLE x.L                     =   6.99667221
             VARIABLE z.L                     =  65.49131329

----       9 PARAMETER phi                    =  1.000000E+2
             VARIABLE x.L                     =   6.99966672
             VARIABLE z.L                     =  65.07214719

----       9 PARAMETER phi                    =  1.000000E+3
             VARIABLE x.L                     =   6.99996667
             VARIABLE z.L                     =  65.00951720

----       9 PARAMETER phi                    =  1.000000E+4
             VARIABLE x.L                     =   6.99999667
             VARIABLE z.L                     =  65.00118198
```

GAMS

$$\min_x \ [5(x-10)^2 - 20] - \left[\frac{1}{\phi}\log(-(1-x)) + \frac{1}{\phi}\log(-(x-7))\right]$$

```
----      9 PARAMETER phi                  =  1.000000E+5
            VARIABLE x.L                   =   6.99999967
            VARIABLE z.L                   =  65.00014122

----      9 PARAMETER phi                  =  1.000000E+6
            VARIABLE x.L                   =   6.99999997
            VARIABLE z.L                   =  65.00001642

----      9 PARAMETER phi                  =  1.000000E+7
            VARIABLE x.L                   =   7.00000000
            VARIABLE z.L                   =  65.00000187
```

Different than 25!

This is it!