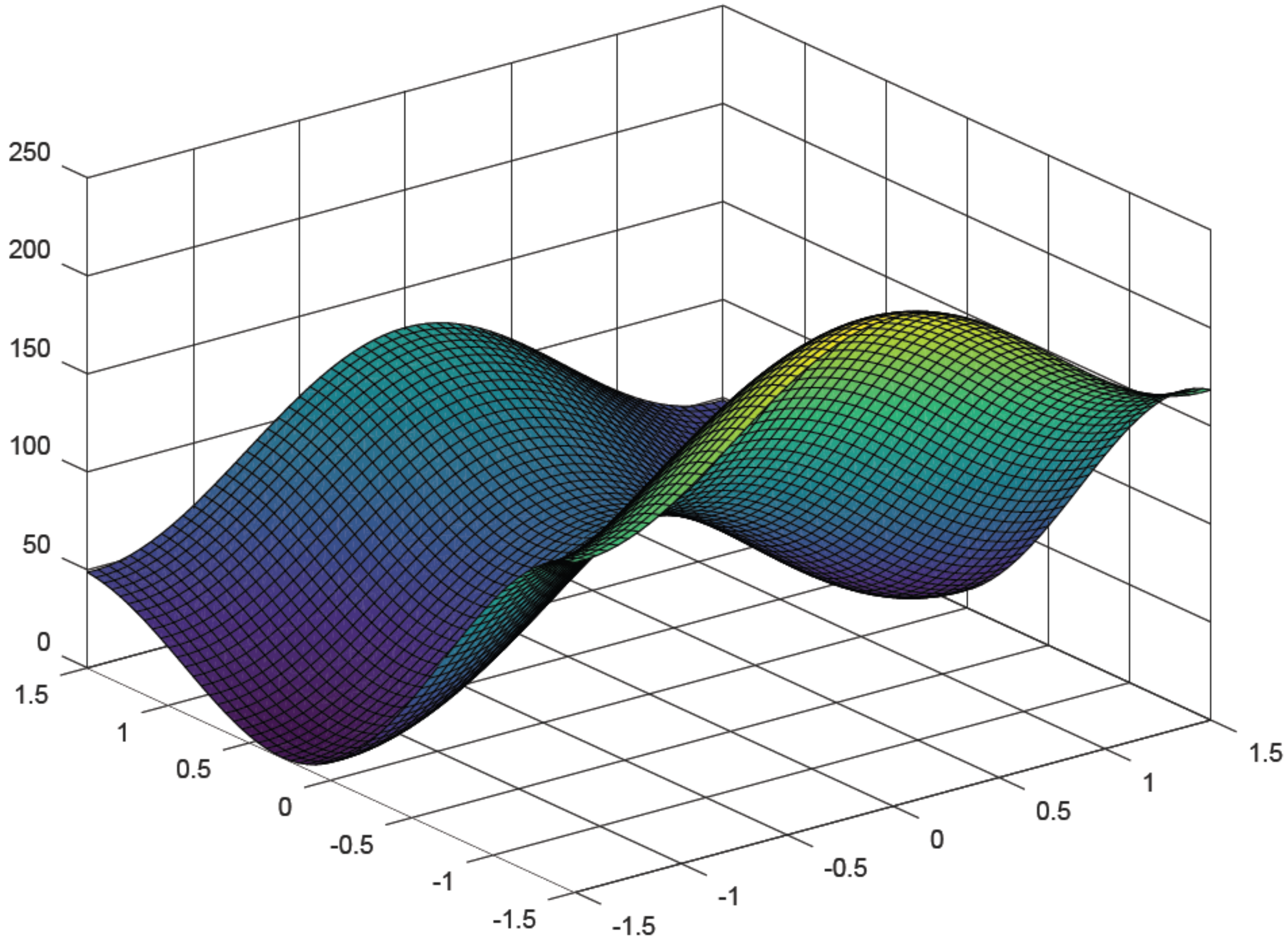
NLP: Examples



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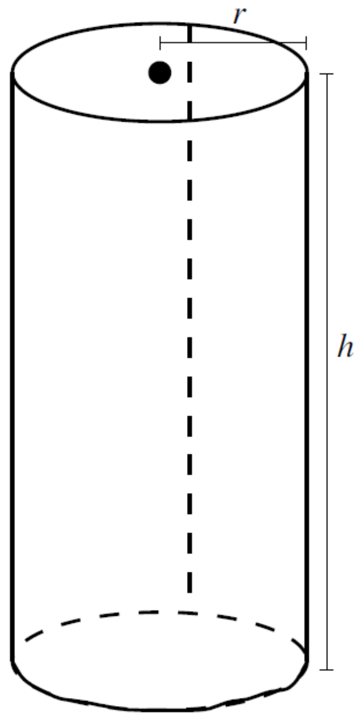
The Ohio State University

What

1. Cylinder Problem
2. Awning Problem
3. Packing-Box Problem
4. Optimal Power Flow Problem

Cylinder Problem

Cylinder Problem

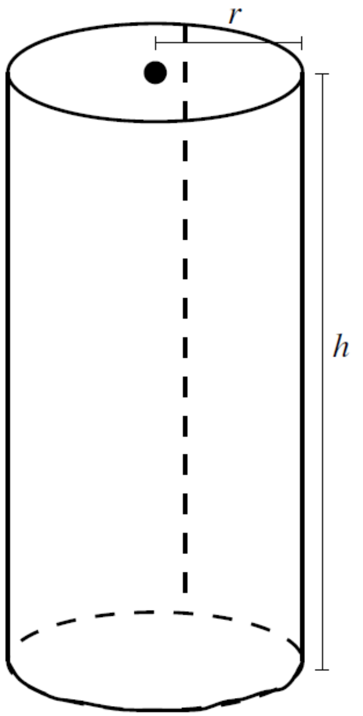


A brewery would like to design a cylindrical vat (see figure below), in which to brew beer.

The vat is h m high and has a radius of r m.

The material used to construct the top of the vat costs $\$c_1$ per m^2.

Cylinder Problem



The material cost of the side and bottom of the vat is proportional to the volume of the vat.

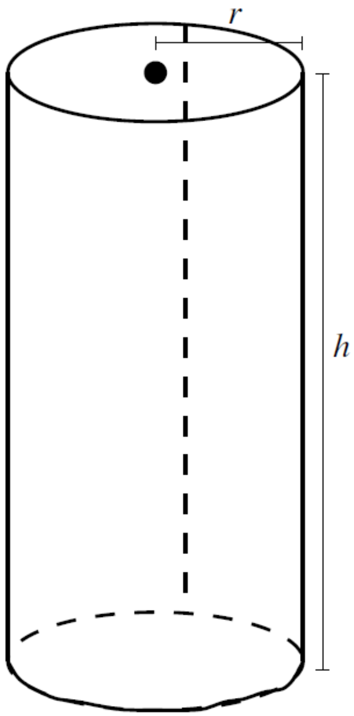
This is because the side and bottom must be reinforced to hold the volume of liquid that is brewed in the vat.

This material cost is \$$c_2 V$ per m^2, where V is the volume of the vat.

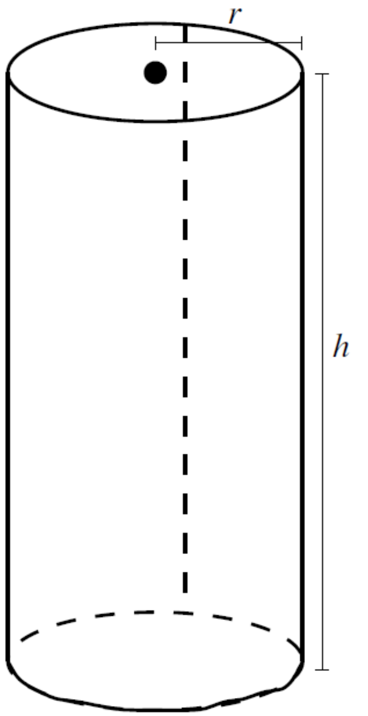
Cylinder Problem

The value of the beer that is produced in the vat during its usable life is proportional to the volume of the vat, and is given by \$$N$ per m^3.

The brewery would like to design the vat to maximize the net profit earned over its usable life.



Cylinder Problem



This NLPP has two decision variables, h and r, which measure the height and radius, respectively, of the vat in meters.

Cylinder Problem

The objective is to maximize the value of the beer brewed less the cost of building the vat.

The value of the beer produced by the vat is given by:

$$N(\pi r^2 h).$$

The cost of top of the vat is:

$$c_1(\pi r^2).$$

Cylinder Problem

The per-unit material cost (in \$/m^3) of the side and bottom of the vat is given by:

$$c_2(\pi r^2 h).$$

Thus the bottom of the vat costs:

$$(c_2 \pi r^2 h)\pi r^2,$$

and the side costs:

$$(c_2 \pi r^2 h)2\pi r h.$$

Cylinder Problem

Thus, the objective, which is to maximize the value of the beer produced by the vat less its material costs, is:

$$\max_{h,r} \; N\pi r^2 h - c_1 \pi r^2 - c_2 \pi r^2 h \cdot (\pi r^2 + 2\pi r h).$$

Cylinder Problem

There is one type of constraint, which is to ensure that the vat has non-negative dimensions:

$$r, h \geq 0.$$

Cylinder Problem

Putting all of this together, the NLPP can be written as:

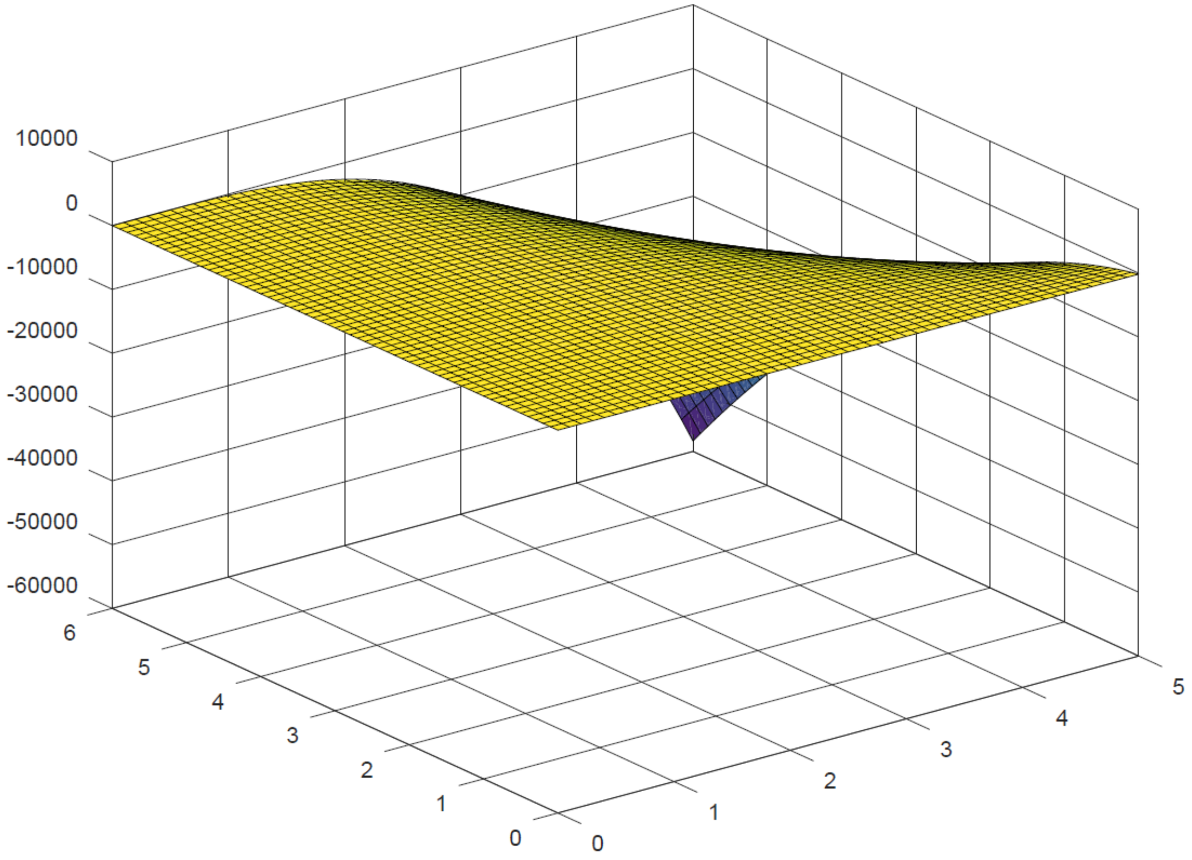
$$\max_{h,r} N\pi r^2 h - c_1 \pi r^2 - c_2 \pi r^2 h \cdot (\pi r^2 + 2\pi r h)$$

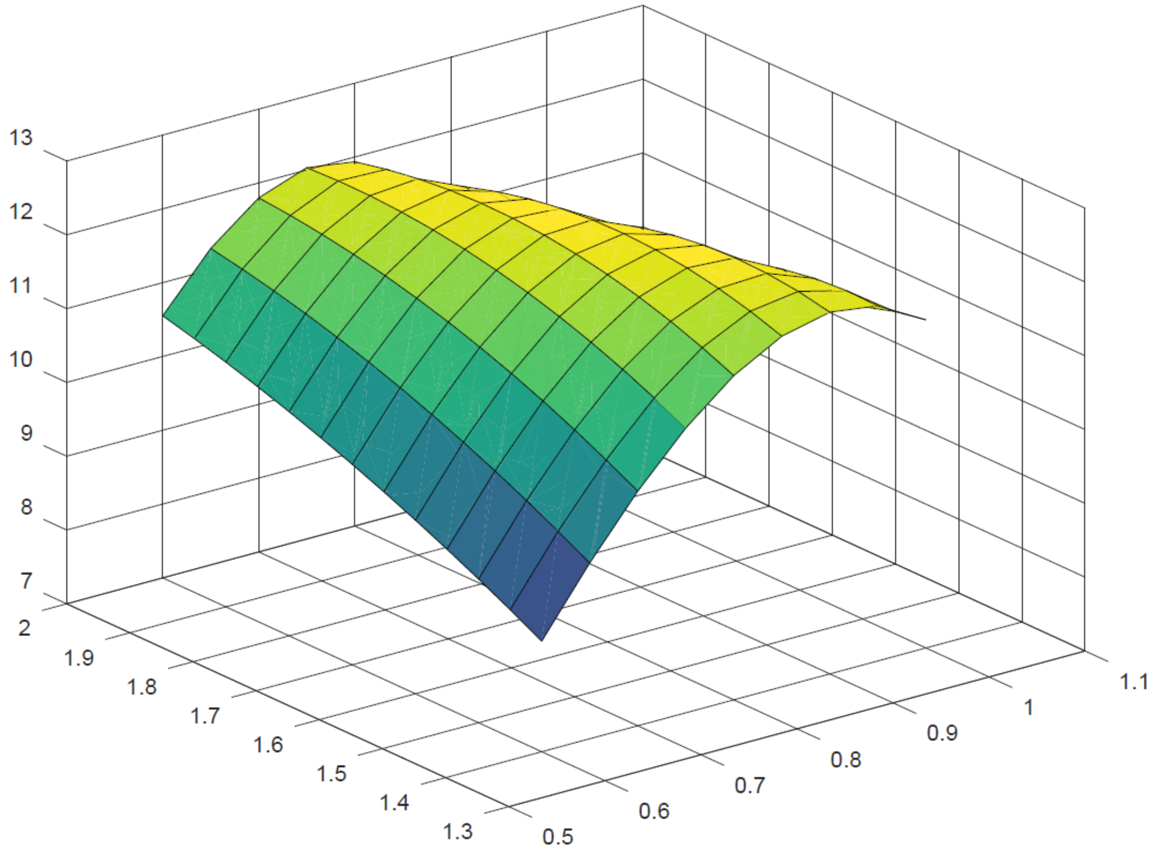
$$\text{s.t.} \quad r, h \geq 0.$$

Cylinder Problem

```
r = -2:1/10:3;
h =  0:1/10:4;
[R,H] = meshgrid(r,h);
F = 10.*pi.*R.^2.*H .- ...
     2.*pi.*R.^2 .- ...
     (1/2).*pi.*R.^2.*H.*(pi.*R.^2 + 2.*pi.*R.*H);
figure
surf(R,H,F)
```

Cylinder Problem





Cylinder Problem

$$\max_{h,r} \; N\pi r^2 h - c_1 \pi r^2 - c_2 \pi r^2 h \cdot (\pi r^2 + 2\pi r h)$$
$$\text{s.t.} \quad r, h \geq 0.$$

Cylinder Problem

$$\max_{h,r} \; z = N\pi r^2 h - c_1 \pi r^2 - c_2 \pi r^2 h \cdot (\pi r^2 + 2\pi r h)$$

$$\text{s.t.} \quad r, h \geq 0.$$

```
scalars N /10/, c1 /2/, c2 /0.5/;
variable z;
positive variables r, h;
r.l=1; h.l=1;
equations of;
of .. z =e= N*pi*h*r**2-c1*pi*r**2-c2*pi*h*(pi*r**2+2*pi*r*h)*r**2;
model cyl /all/;
solve cyl using nlp maximizing z;
```

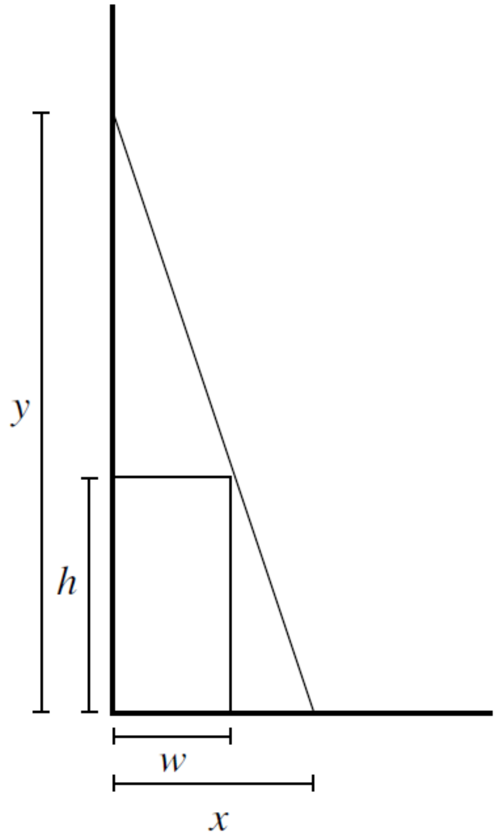
Cylinder Problem

```
                    LOWER     LEVEL     UPPER    MARGINAL

---- VAR z           -INF     12.174     +INF       .
---- VAR r             .       0.826     +INF      EPS
---- VAR h             .       1.721     +INF      EPS
```

Awning Problem

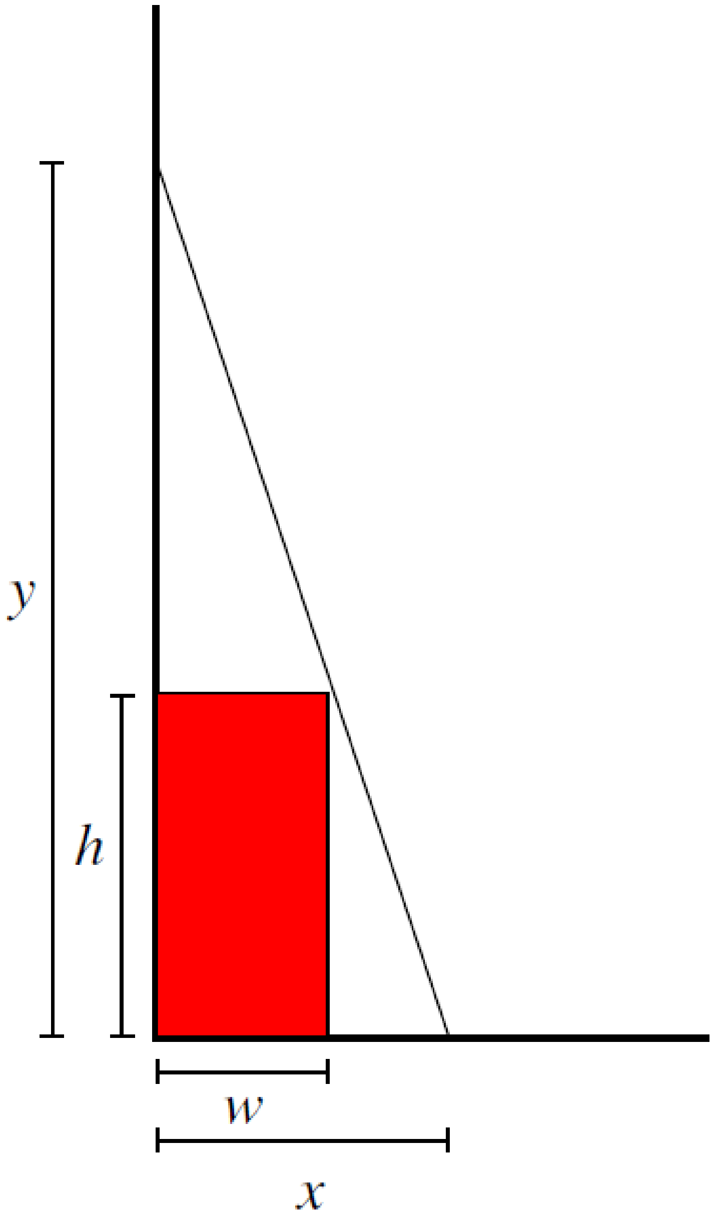
Awning Problem



A box, which is h m high and w m wide is placed against the side of a building (see figure below).

The building owner would like to construct an awning of minimum length that completely covers the box.

Awning Problem

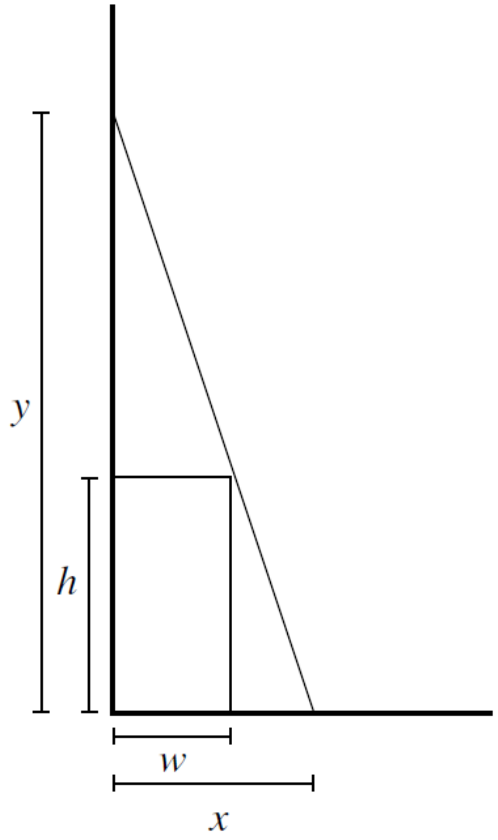


Awning Problem

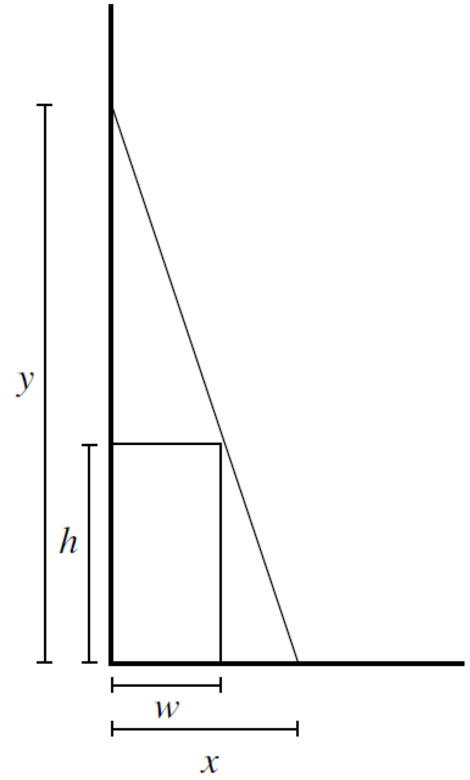
There are two decision variables in this problem.

The first, x, measures the distance between the building wall and the point at which the awning is anchored to the ground, in meters.

The second, y, measures the height of the anchor point of the awning to the building wall, also in meters.



Awning Problem



The objective is to minimize the length of the awning. From the Pythagorean theorem this is:

$$\min_{x,y} \sqrt{x^2 + y^2}.$$

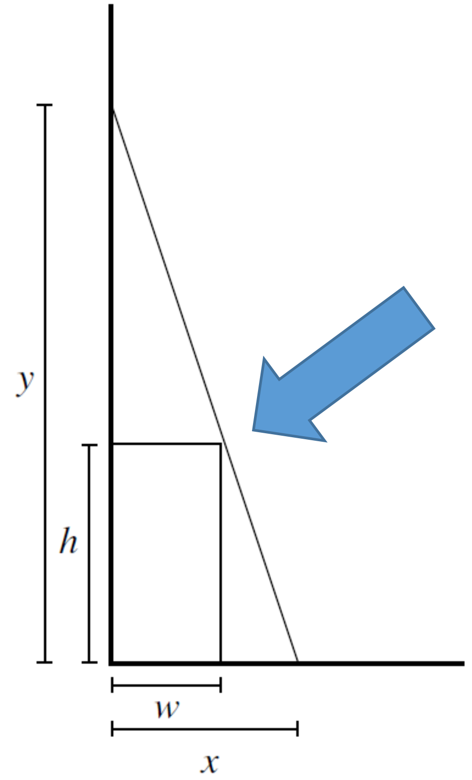
Awning Problem

We have two types of constraints in this problem.

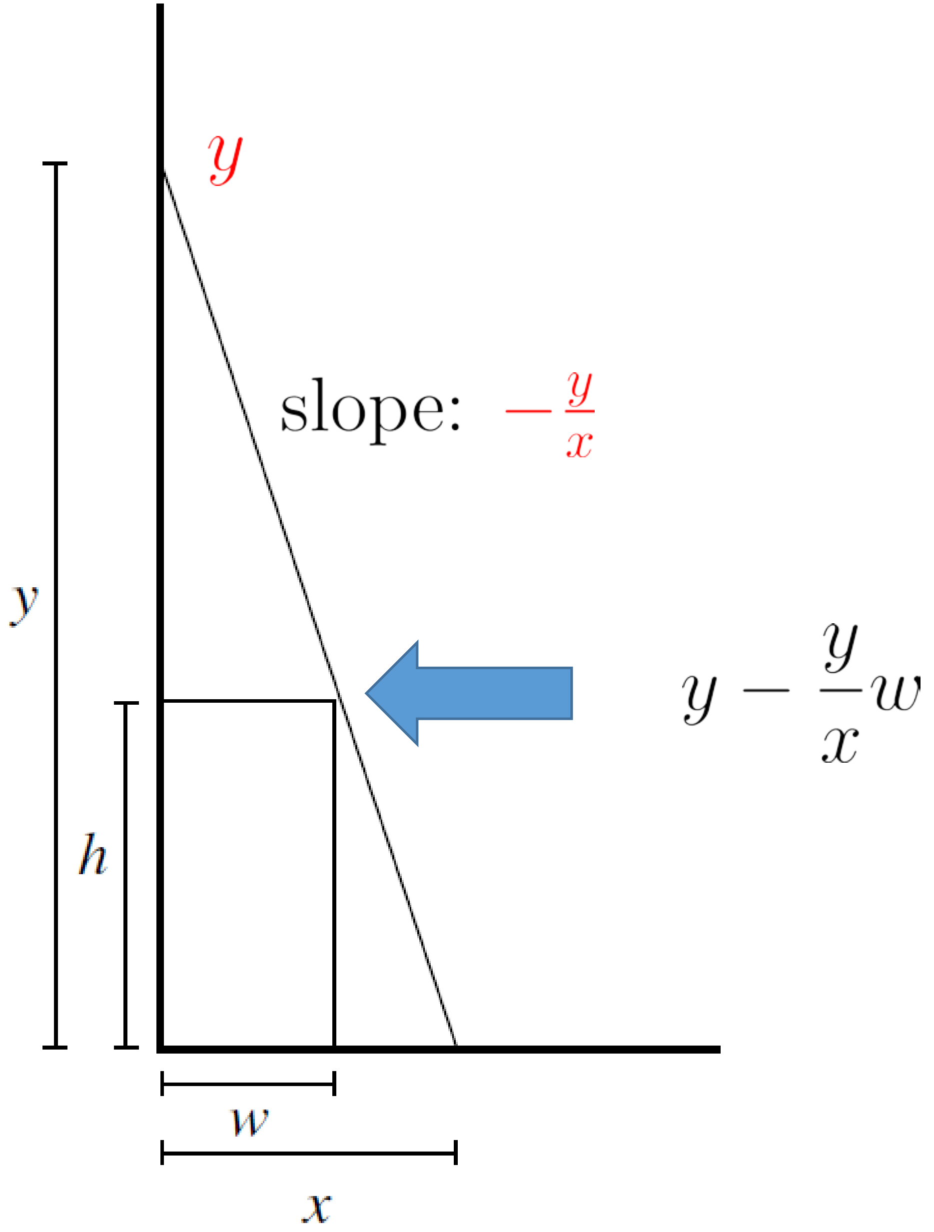
The first ensures that the upper-right corner of the box is below the awning (which ensures that the box is wholly contained by the awning).

To derive this constraint we compute the height of the awning w m away from the wall as:

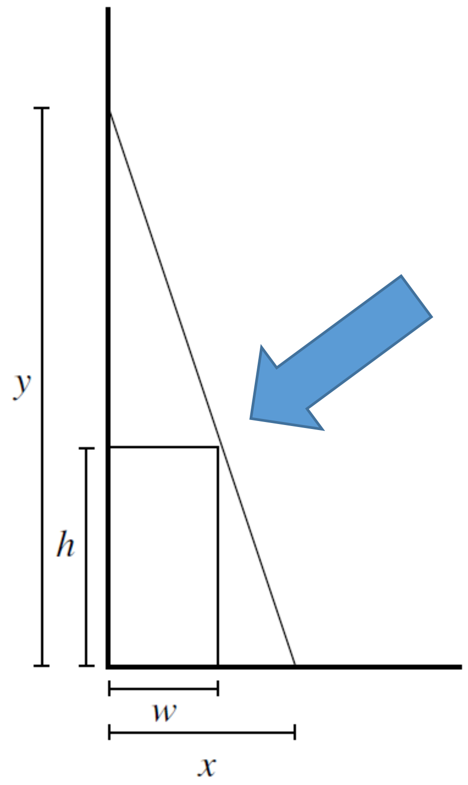
$$y - \frac{y}{x} w.$$



Awning Problem



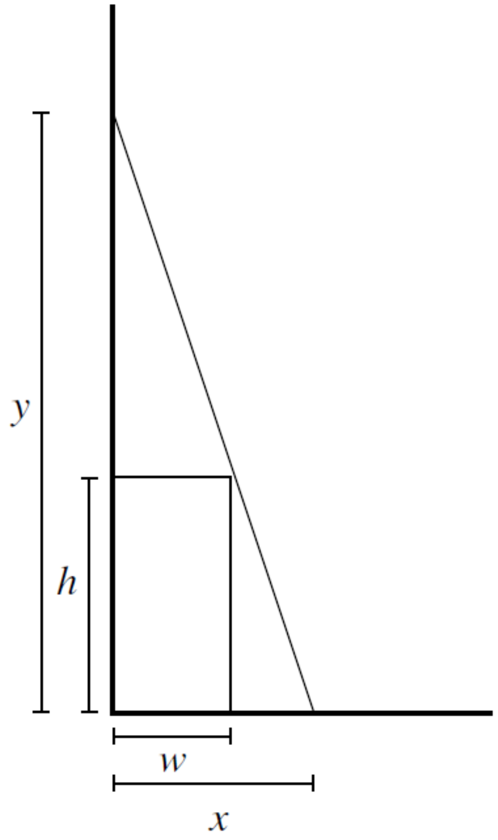
Awning Problem



To ensure that the upper-right corner of the box is below the awning, this point must be at least h m high, giving our first constraint:

$$y - \frac{y}{x}w \geq h.$$

Awning Problem



We must also ensure that the distances between the anchor points of the awning and the building and ground are non-negative:

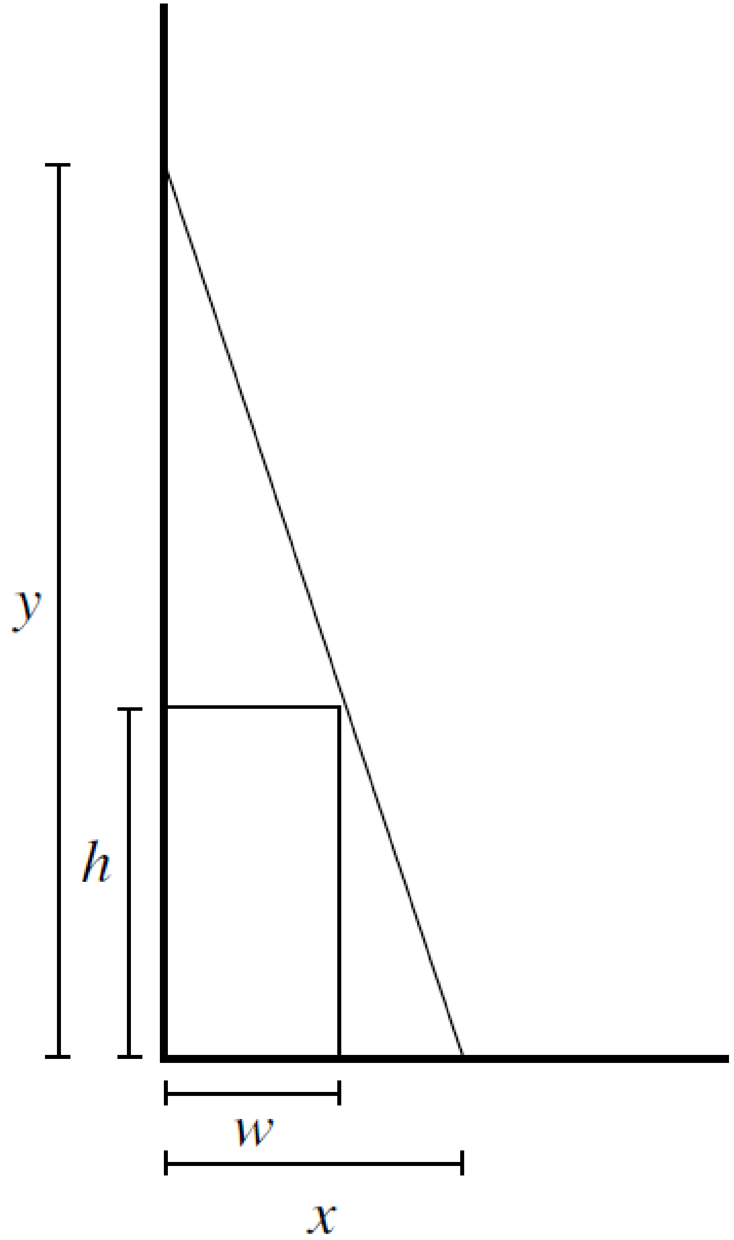
$$x, y \geq 0.$$

Awning Problem

$$\min_{x,y} \; z = \sqrt{x^2 + y^2}$$

$$\text{s.t.} \quad y - \frac{y}{x} w \geq h$$

$$x, y \geq 0.$$



Awning Problem

$$\min_{x,y} \ z = \sqrt{x^2 + y^2}$$

$$\text{s.t.} \quad y - \frac{y}{x} w \geq h$$

$$x, y \geq 0.$$

```
scalars h /2/, w /3/;
variable z;
positive variables x, y;
x.l=1; y.l=1;
equations of, box;
of .. z =e= sqrt(x**2+y**2);
box .. y-w*y/x  =g= h;
model awning /all/;
solve awning using nlp minimizing z;
```

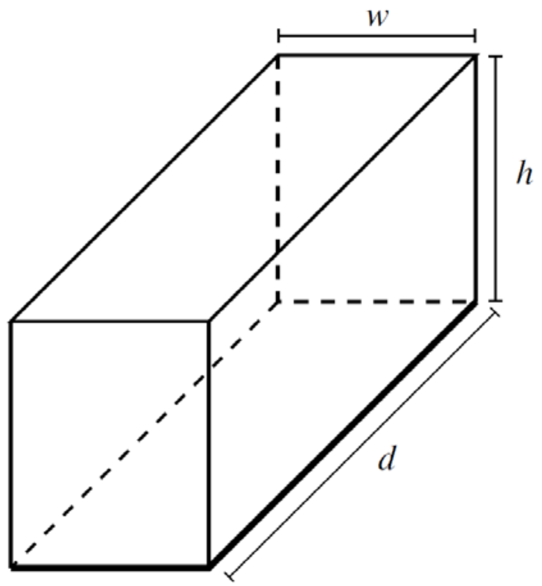
Awning Problem

```
                    LOWER     LEVEL     UPPER    MARGINAL

---- VAR z           -INF     7.023     +INF        .
---- VAR x             .      5.289     +INF        .
---- VAR y             .      4.621     +INF       EPS
```

Packing-Box Problem

Packing-Box Problem

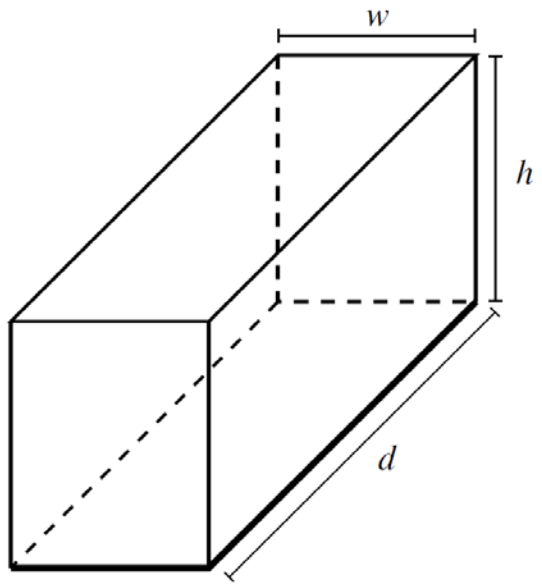


A company must determine the dimensions of a cardboard box to maximize its volume.

The box can use at most 60 cm^2 of cardboard.

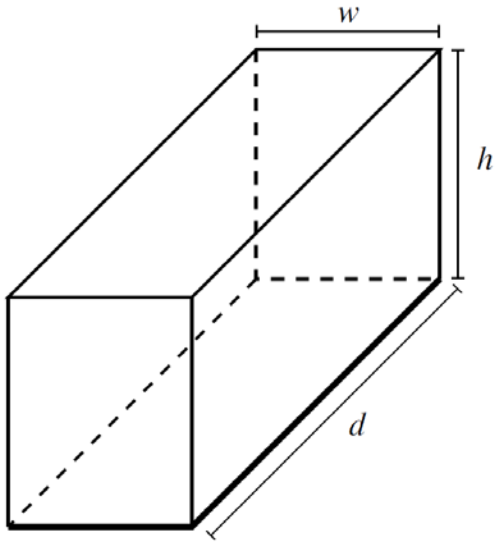
For structural reasons, the bottom and top faces of the box must be of triple weight (i.e., three pieces of cardboard).

Packing-Box Problem



There are three decision variables in this problem, h, w, and h, which are the height, width, and depth of the box in cm, respectively (see figure below).

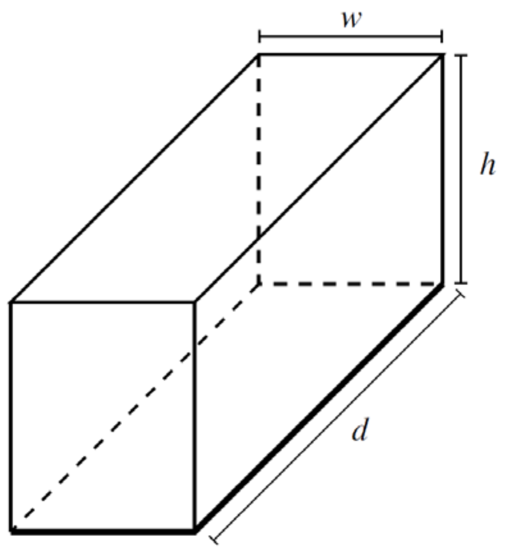
Packing-Box Problem



The objective is to maximize the volume of the box:

$$\max_{h,w,d} \ hwd.$$

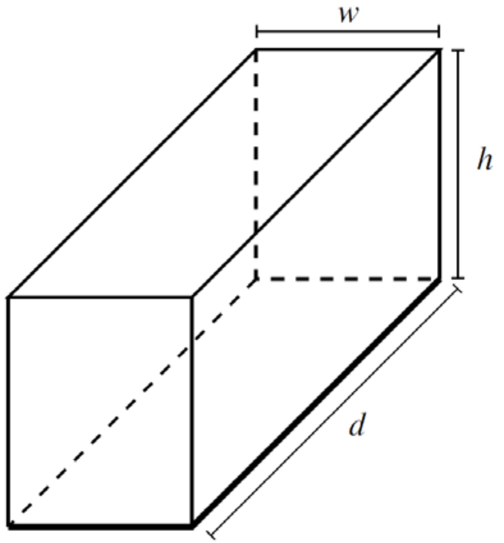
Packing-Box Problem



There are two types of constraints. First, we must ensure that the box uses no more than 60 cm^2 of cardboard, noting that the top and bottom of the box are of triple weight:

$$2wh + 2dh + 6wd \leq 60.$$

Packing-Box Problem



The second type of constraint ensures that the dimensions of the box are non-negative, as negative dimensions are physically impossible:

$$w, h, d \geq 0.$$

Packing-Box Problem

Putting all of this together, the NLPP can be written as:

$$\begin{aligned} \max_{h,w,d} \quad & z = hwd \\ \text{s.t.} \quad & 2wh + 2dh + 6wd \leq 60 \\ & w, h, d \geq 0. \end{aligned}$$

Packing-Box Problem

$$\max_{h,w,d} \quad z = hwd$$

$$\text{s.t.} \quad 2wh + 2dh + 6wd \leq 60$$

$$w, h, d \geq 0.$$

```
variable z;
positive variables h, w, d;
h.l=1; w.l=1; d.l=1;
equations of, cardBoard;
of .. z =e= h*w*d;
cardBoard .. 2*w*h+2*h*d+6*w*d =l= 60;
model box /all/;
Option NLP = KNITRO;
solve box using nlp maximizing z;
```

Packing-Box Problem

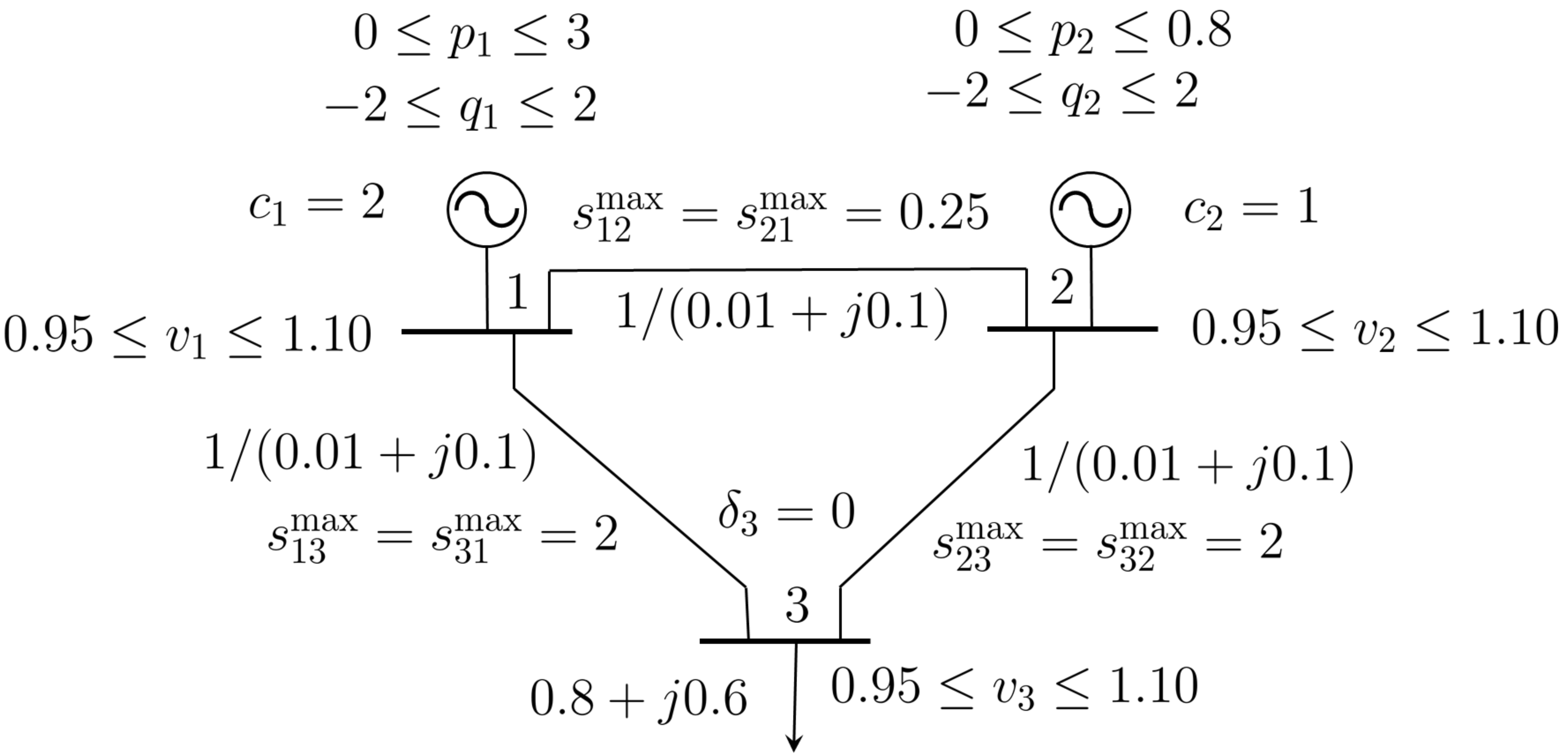
	LOWER	LEVEL	UPPER	MARGINAL
---- VAR z	-INF	18.257	+INF	.
---- VAR h	.	5.477	+INF	EPS
---- VAR w	.	1.826	+INF	-2.31E-11
---- VAR d	.	1.826	+INF	-2.31E-11

Optimal power flow

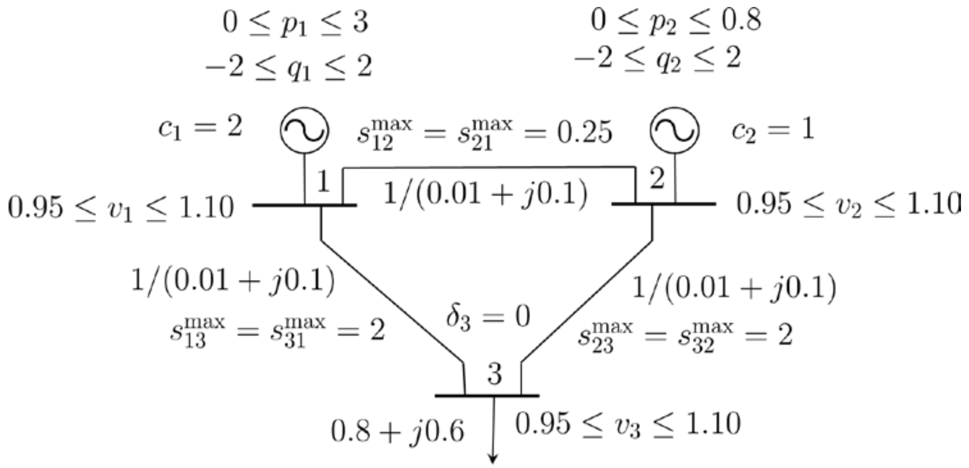
Optimal power flow

1. Optimal power flow: Introduction
2. Formulation
3. Example
4. GAMS code

Introduction



Introduction



The objective of the optimal power flow problem is

to find out the power output of every unit (including both active and reactive power)

so that all loads are supplied at minimum cost (other objectives are also possible),

while satisfying network constraints.

Introduction

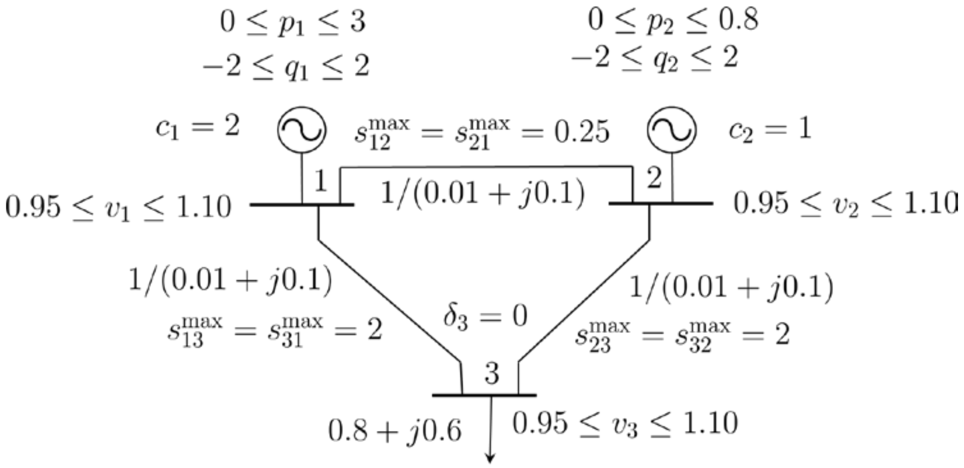
We cannot guarantee this with a power flow

This is a security issue!

Particularly,

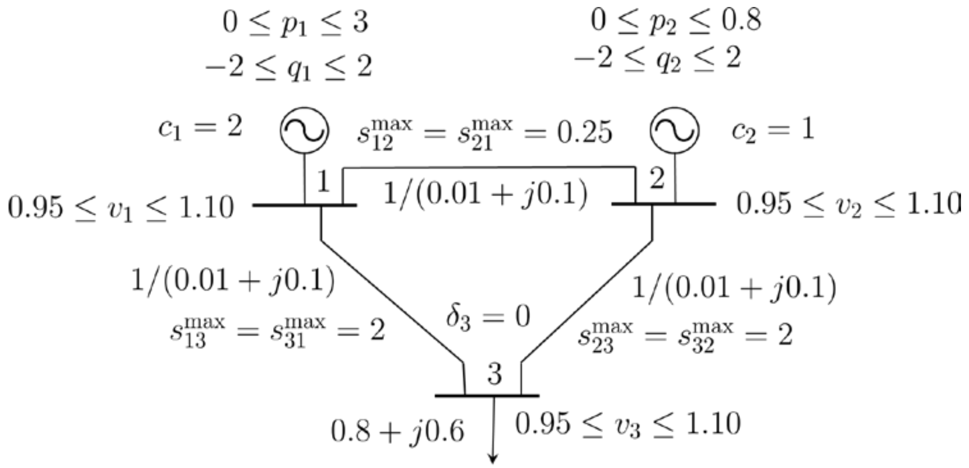
transmission lines should not be overloaded and

voltage magnitudes throughout the network should be at acceptable levels.



Formulation

Formulation



The active power $p_{ij}(\cdot)$ from node i to j through line ij is

$$p_{ij}(\cdot) = v_i^2 y_{Lij} \cos(\theta_{Lij}) - v_i v_j y_{Lij} \cos(\delta_i - \delta_j - \theta_{Lij}) + \frac{1}{2} v_i^2 y_{Sij} \cos(\theta_{Sij}),$$

Formulation

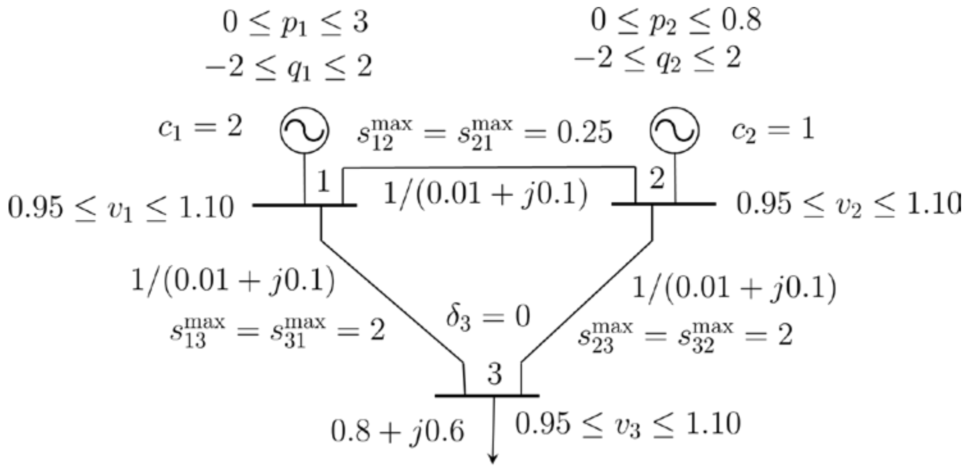
where

$v_i \angle \delta_i$ and $v_j \angle \delta_j$ are the voltages at nodes i and j, respectively,

$y_{Lij} \angle \theta_{Lij}$ the per unit admittance of line ij, and

$y_{Sij} \angle \theta_{Sij}$ the per unit shunt admittance of line ij.

Formulation

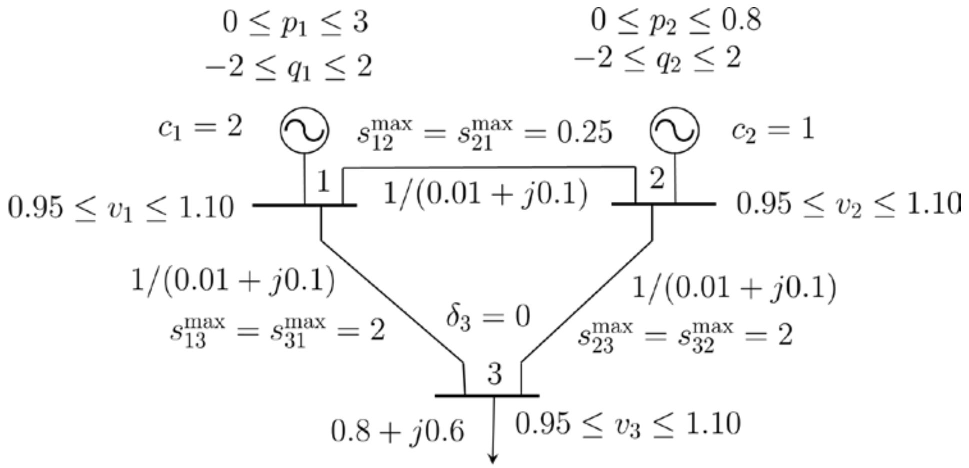


Active power balance at any node needs to be imposed, thus,

$$\sum_{g \in \Omega_i} p_g - d_i^{\mathrm{P}} = \sum_{k \in \Lambda_i} p_{ik}(\cdot) \quad \forall i = 1, \ldots, n, \qquad (1)$$

where Ω_i is the set of units at node i, Λ_i the set of nodes directly connected to node i, and d_i^{P} the active power demand at node i.

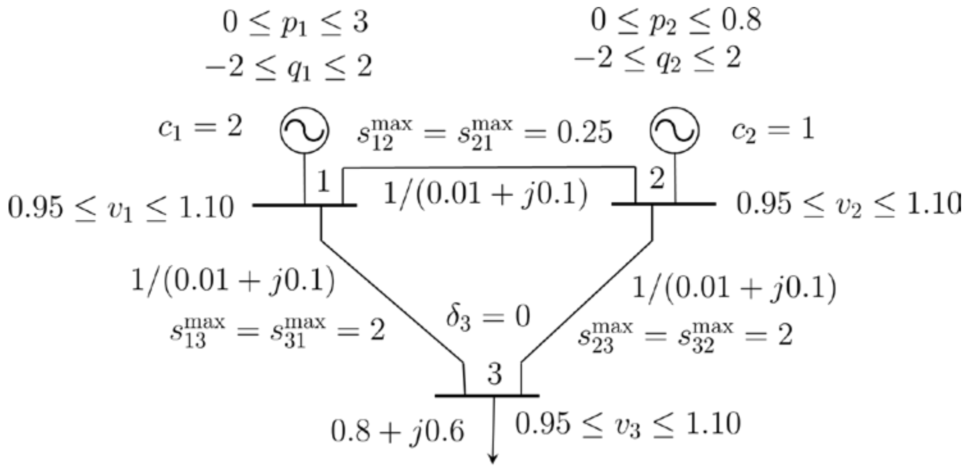
Formulation



The reactive power $q_{ij}(\cdot)$ from node i to j through line ij is

$$q_{ij}(\cdot) = -v_i^2 y_{Lij} \sin(\theta_{Lij}) - v_i v_j y_{Lij} \sin(\delta_i - \delta_j - \theta_{Lij}) - \frac{1}{2} v_i^2 y_{Sij} \sin(\theta_{Sij}).$$

Formulation

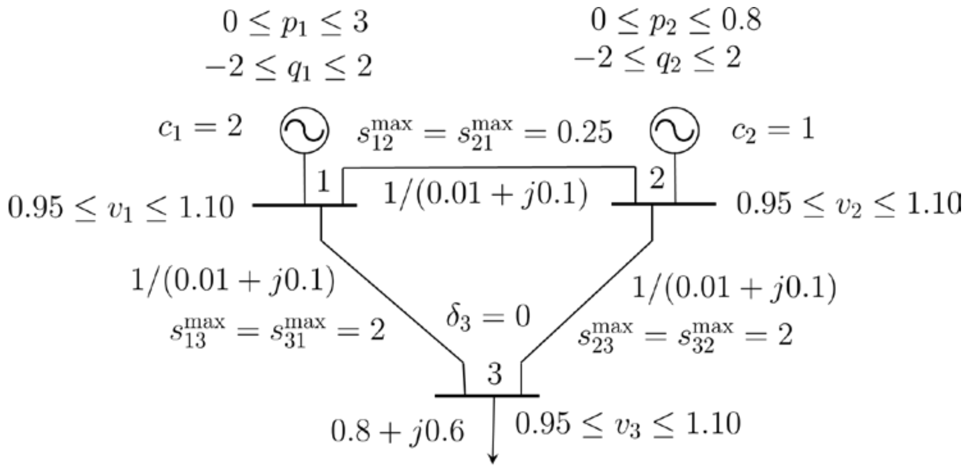


Reactive power balance at any node needs to be imposed as well, thus,

$$\sum_{g \in \Omega_i} q_g - d_i^{\mathrm{Q}} = \sum_{k \in \Lambda_i} q_{ik}(\cdot) \quad \forall i = 1, \ldots, n, \qquad (2)$$

where d_i^{Q} the reactive power demand at node i.

Formulation

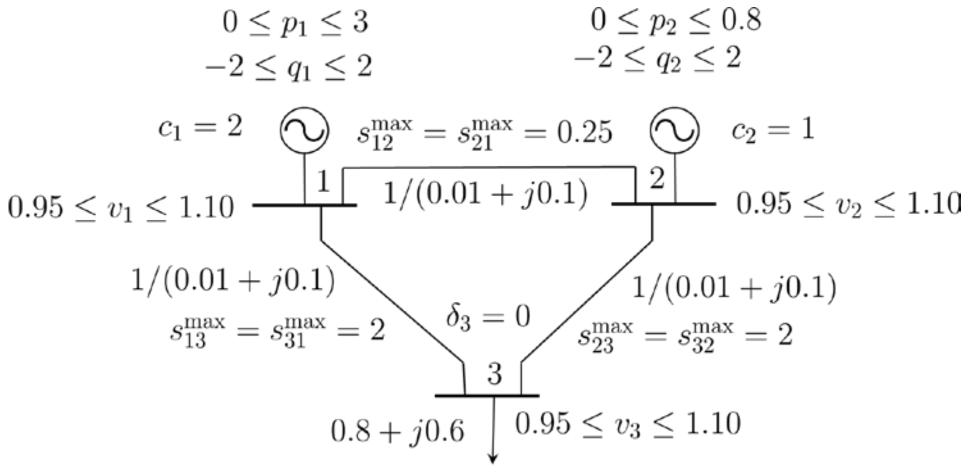


The voltage magnitude of any bus i is limited above and below, therefore,

$$v_i^{\min} \leq v_i \leq v_i^{\max}, \tag{3}$$

where $v_i^{\min}$ and $v_i^{\max}$ are respectively the lower and upper bounds of the voltage magnitude at bus i.

Formulation

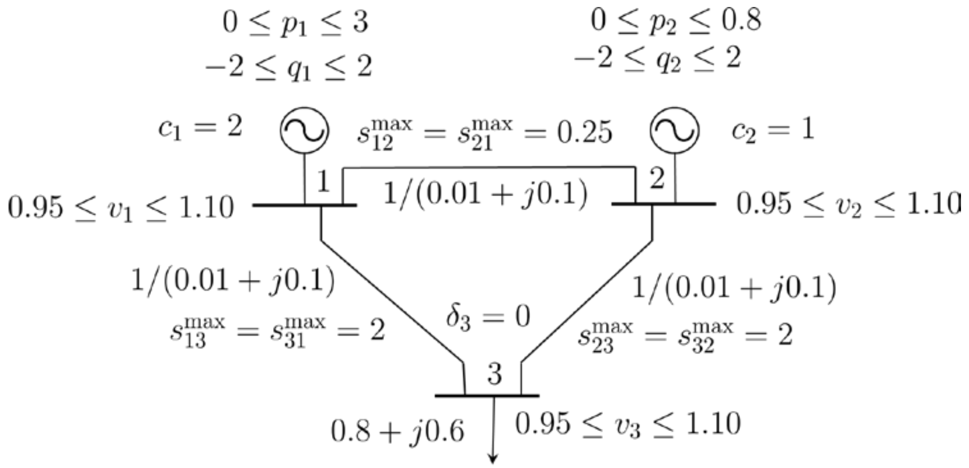


Any unit g can produce active power above and below a lower and an upper limit, respectively, i.e.,

$$p_g^{\min} \le p_g \le p_g^{\max} \tag{4}$$

where $p_g^{\min}$ is the minimum active power output and $p_g^{\max}$ the maximum.

Formulation

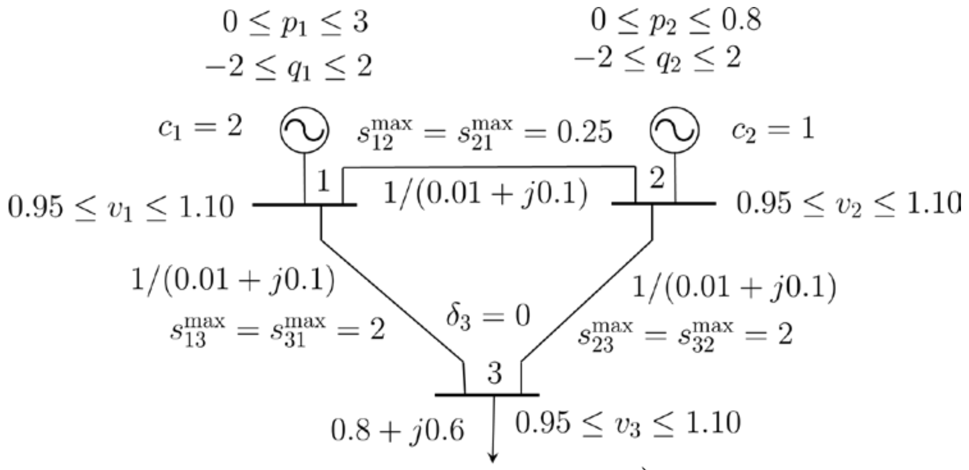


Similarly, any unit g can produce reactive power above and below a lower and an upper limit, respectively, i.e.,

$$q_g^{\min} \le q_g \le q_g^{\max}, \tag{5}$$

where $q_g^{\min}$ is the minimum reactive power output and $q_g^{\max}$ the maximum.

Formulation

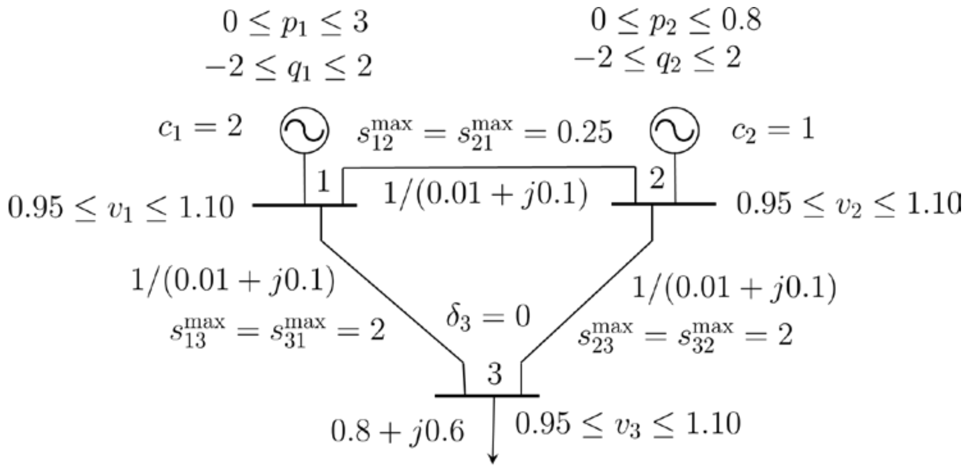


The apparent power (magnitude of the complex power) from node i to j through line ij

$$s_{ij}(\cdot) = +\sqrt{(p_{ik}(\cdot))^2 + (q_{ik}(\cdot))^2}$$

is limited in each line, thus

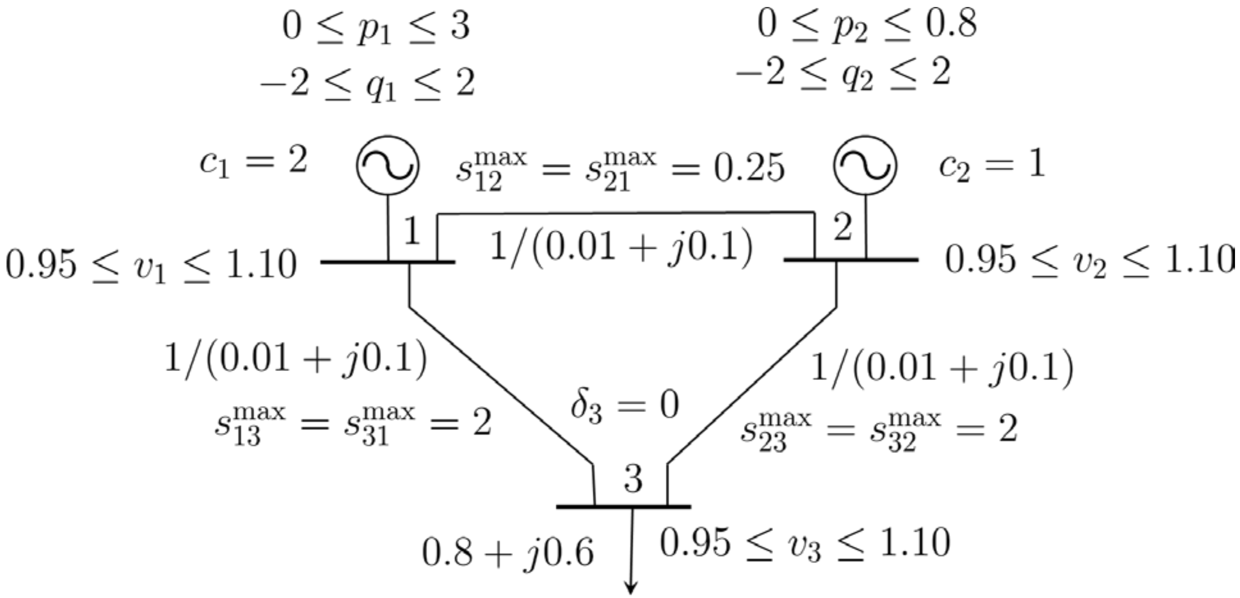
Formulation



$$+\sqrt{(p_{ik}(\cdot))^2 + (q_{ik}(\cdot))^2} \leq s_{ik}^{\max}, \qquad (6)$$

where $s_{ik}^{\max}$ if the maximum apparent power through line ij.

Formulation

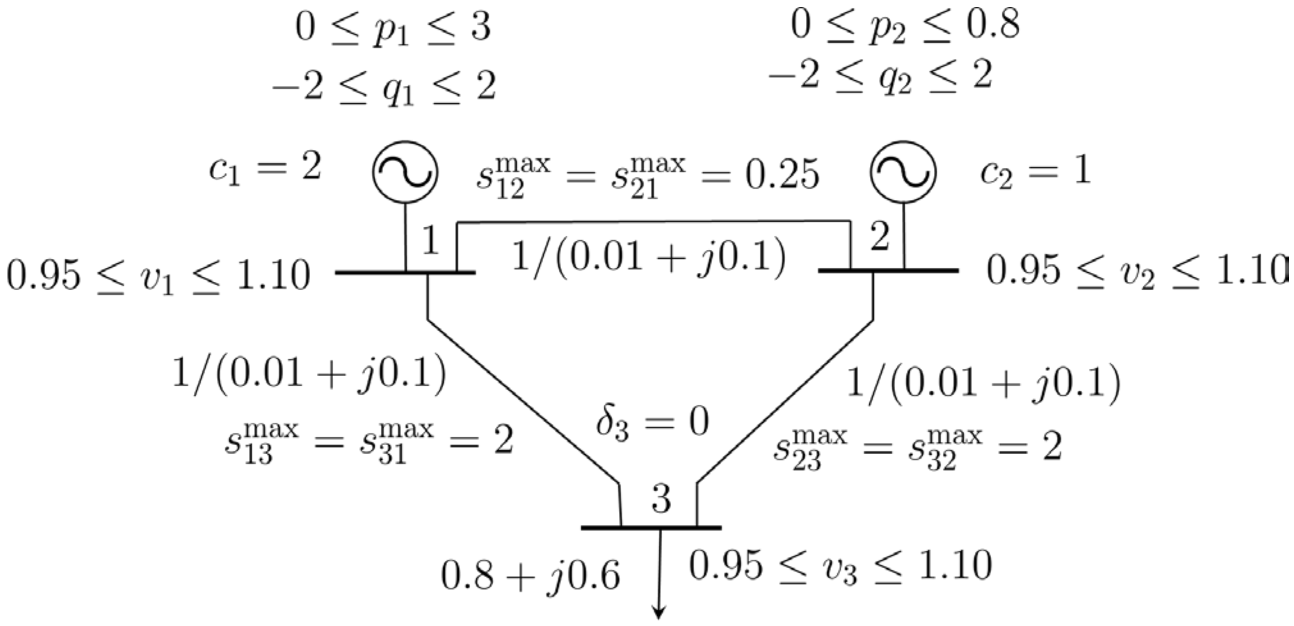


If c_i is the marginal cost of active power production by unit g, the total production cost is

$$\sum_{g \in \Omega} c_g p_g, \qquad (7)$$

where Ω is the set of all production units.

Formulation



There is no significant cost for reactive power production.

Formulation

Considering the above, the optimal power flow can then be formulated as

Formulation

$$\min_{\Xi} \sum_{g\in\Omega} c_g p_g \tag{8a}$$

s.t.

$$\sum_{g\in\Omega_i} p_g - d_i^{\mathrm{P}} = \sum_{k\in\Lambda_i} p_{ik}(\cdot) \quad \forall i = 1,\ldots,n \tag{8b}$$

$$\sum_{g\in\Omega_i} q_g - d_i^{\mathrm{Q}} = \sum_{k\in\Lambda_i} q_{ik}(\cdot) \quad \forall i = 1,\ldots,n \tag{8c}$$

$$+\sqrt{(p_{ik}(\cdot))^2 + (q_{ik}(\cdot))^2} \leq s_{ik}^{\max} \quad \forall i = 1,\ldots,n, \forall k \in \Lambda_i \tag{8d}$$

$$p_g^{\min} \leq p_g \leq p_g^{\max}, \quad \forall g \in \Omega \tag{8e}$$

$$q_g^{\min} \leq q_g \leq q_g^{\max}, \quad \forall g \in \Omega \tag{8f}$$

$$v_i^{\min} \leq v_i \leq v_i^{\max}, \quad \forall i = 1,\ldots,n \tag{8g}$$

$$-\pi \leq \delta_i \leq \pi, \quad \forall i = 1,\ldots,n \tag{8h}$$

$$\delta_l = 0, \quad l = \text{reference node} \tag{8i}$$

Formulation

where

$$\Xi = \{v_1, \ldots, v_n, \delta_1, \ldots, \delta_n; p_g, q_g, \forall g \in \Omega\}$$

is the set of optimization variables.

Formulation

And

$$p_{ij}(\cdot) = v_i^2 y_{Lij} \cos(\theta_{Lij}) - v_i v_j y_{Lij} \cos(\delta_i - \delta_j - \theta_{Lij}) + \frac{1}{2} v_i^2 y_{Sij} \cos(\theta_{Sij}),$$

$$q_{ij}(\cdot) = -v_i^2 y_{Lij} \sin(\theta_{Lij}) - v_i v_j y_{Lij} \sin(\delta_i - \delta_j - \theta_{Lij}) - \frac{1}{2} v_i^2 y_{Sij} \sin(\theta_{Sij}).$$

Formulation

Objective function (8a) is the total cost,

constraints (8b) are active power balance per node,

constraints (8c) are reactive power balance per node,

constraints (8d) enforce line capacity limit per line,

constraints (8e) enforce active power production limits per unit,

constraints (8f) enforce reactive power production limits per unit,

constraints (8g) enforce voltage limits per node, and

constraint (8h) sets the reference node.

Formulation

Problem (8) is denominated optimal power flow.

Example

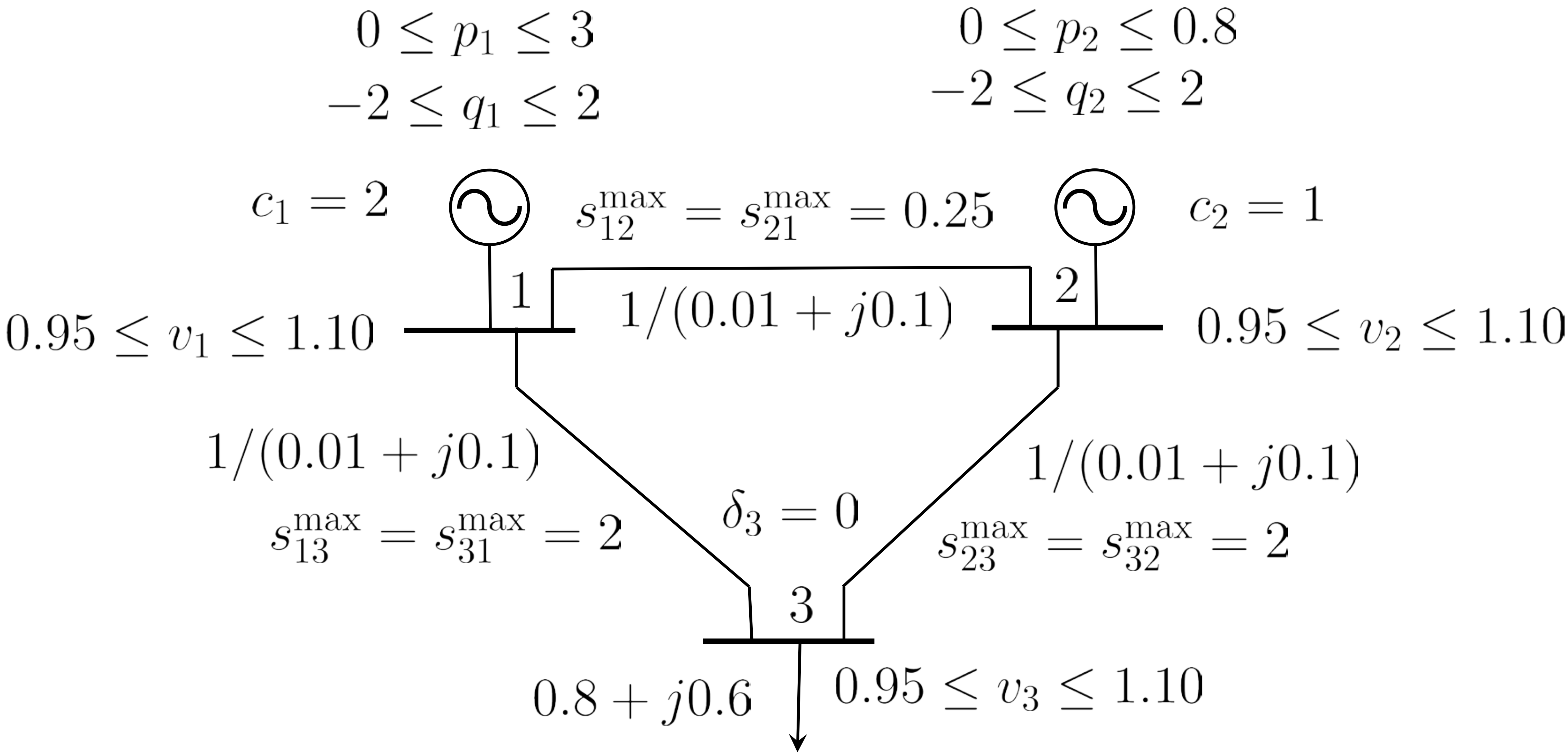
Example

Consider a power network with

3 nodes and 3 lines as depicted in the figure below.

Network parameters in per unit and radians are given below as well.

Example


$0 \le p_1 \le 3$
$-2 \le q_1 \le 2$
$0 \le p_2 \le 0.8$
$-2 \le q_2 \le 2$
$c_1 = 2$
$s_{12}^{\max} = s_{21}^{\max} = 0.25$
$c_2 = 1$
1
2
$0.95 \le v_1 \le 1.10$
$1/(0.01 + j0.1)$
$0.95 \le v_2 \le 1.10$
$1/(0.01 + j0.1)$
$1/(0.01 + j0.1)$
$\delta_3 = 0$
$s_{13}^{\max} = s_{31}^{\max} = 2$
$s_{23}^{\max} = s_{32}^{\max} = 2$
3
$0.8 + j0.6$
$0.95 \le v_3 \le 1.10$

Example

$$y_{L12}\angle\theta_{L12} = \frac{1}{0.01 + j0.1} = 9.95037190209989\angle - 1.47112767430373$$

$$y_{L13}\angle\theta_{L13} = \frac{1}{0.01 + j0.1} = 9.95037190209989\angle - 1.47112767430373$$

$$y_{L23}\angle\theta_{L23} = \frac{1}{0.01 + j0.1} = 9.95037190209989\angle - 1.47112767430373$$

$$y_{S12}\angle\theta_{S12} = 0\angle\pi/2$$

$$y_{S13}\angle\theta_{S13} = 0\angle\pi/2$$

$$y_{S23}\angle\theta_{S23} = 0\angle\pi/2.$$

Example

Additionally:

1. Node 1 is a generating node with lower and upper bounds on active power generation equal to 0 and 3, respectively, and of reactive power generation equal to -2 and 2, respectively.

Example

2. Node 2 is also a generating node with lower and upper bounds on active power generation equal to 0 and 0.8, respectively, and of reactive power generation equal to -2 and 2, respectively.

Example

3. Node 3 is a load node with a active and reactive power demands equal to 0.8 and 0.6, respectively.

Example

4. Voltage magnitude lower and upper limits for all nodes are 0.95 and 1.10, respectively.

5. Marginal production cost of units 1 and 2 are 2 and 1, respectively.

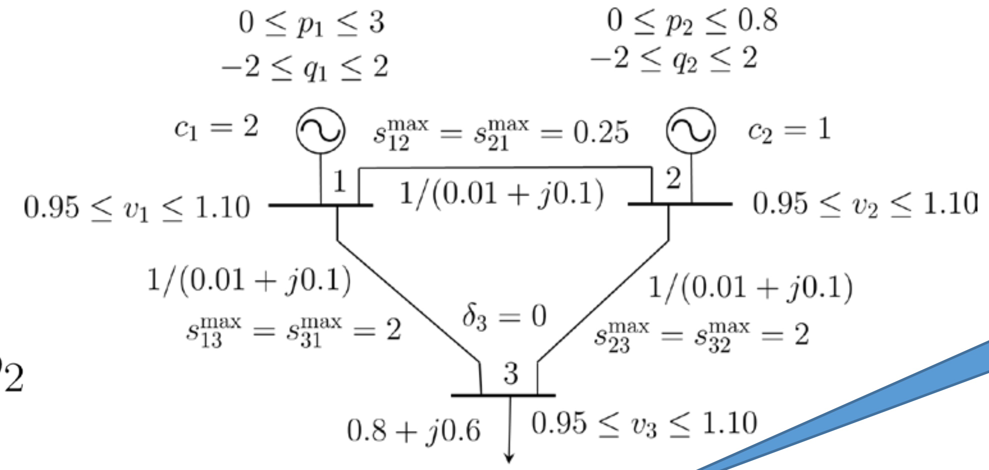
6. The reference node is node 3.

Example

The optimal power flow problem for this example is

$$p_{ij}(\cdot) = v_i^2 y_{Lij}\cos(\theta_{Lij}) - v_i v_j y_{Lij}\cos(\delta_i - \delta_j - \theta_{Lij}) + \frac{1}{2} v_i^2 y_{Sij}\cos(\theta_{Sij})$$

Example



P-balance per node

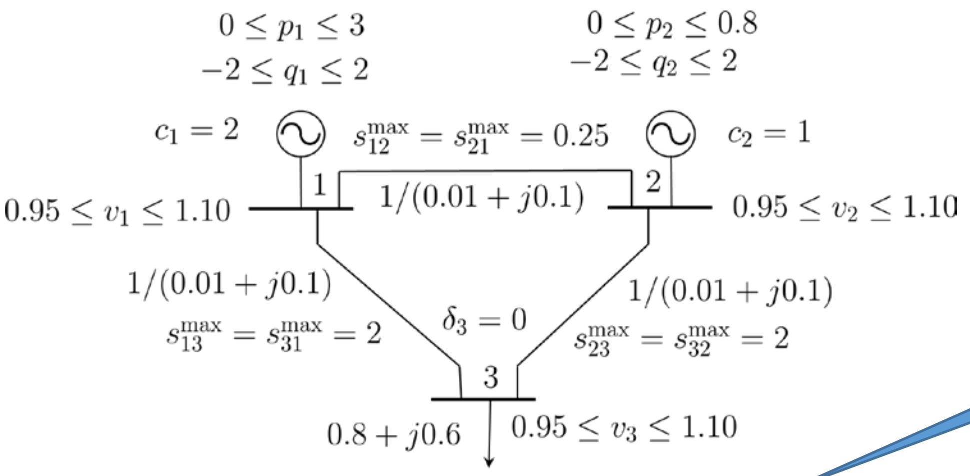
$$\min_{p_1,p_2,q_1,q_2,v_1,v_2,v_3,\delta_1,\delta_2} \quad 2p_1 + 1p_2$$

s.t.

$$p_1 = 9.9504\cos(-1.4711)v_1^2 - 9.9504 v_1 v_2 \cos(\delta_1 - \delta_2 + 1.4711) +$$
$$9.9504\cos(-1.4711)v_1^2 - 9.9504 v_1 v_3 \cos(\delta_1 - 0 + 1.4711)$$
$$p_2 = 9.9504\cos(-1.4711)v_2^2 - 9.9504 v_2 v_1 \cos(\delta_2 - \delta_1 + 1.4711) +$$
$$9.9504\cos(-1.4711)v_2^2 - 9.9504 v_2 v_3 \cos(\delta_2 - 0 + 1.4711)$$
$$-0.8 = 9.9504\cos(-1.4711)v_3^2 - 9.9504 v_3 v_1 \cos(0 - \delta_1 + 1.4711) +$$
$$9.9504\cos(-1.4711)v_3^2 - 9.9504 v_3 v_2 \cos(0 - \delta_2 + 1.4711)$$

$$q_{ij}(\cdot) = -v_i^2 y_{Lij}\sin(\theta_{Lij}) - v_i v_j y_{Lij}\sin(\delta_i - \delta_j - \theta_{Lij}) - \frac{1}{2} v_i^2 y_{Sij}\sin(\theta_{Sij})$$

Example



Q-balance per node

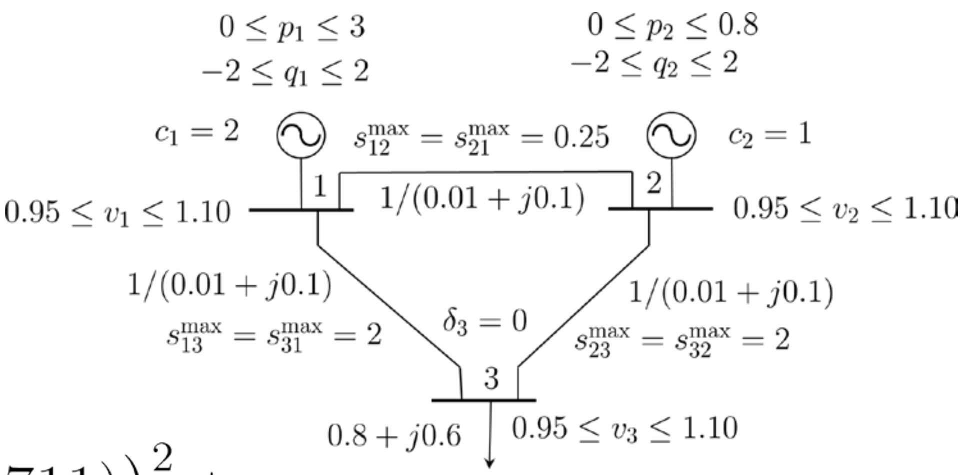
$$q_1 = -9.9504\sin(-1.4711)v_1^2 - 9.9504 v_1 v_2 \sin(\delta_1 - \delta_2 + 1.4711) - 9.9504\sin(-1.4711)v_1^2 - 9.9504 v_1 v_3 \sin(\delta_1 - 0 + 1.4711)$$

$$q_2 = -9.9504\sin(-1.4711)v_2^2 - 9.9504 v_2 v_1 \sin(\delta_2 - \delta_1 + 1.4711) - 9.9504\sin(-1.4711)v_2^2 - 9.9504 v_2 v_3 \sin(\delta_2 - 0 + 1.4711)$$

$$-0.6 = -9.9504\sin(-1.4711)v_3^2 - 9.9504 v_3 v_1 \sin(0 - \delta_1 + 1.4711) - 9.9504\sin(-1.4711)v_3^2 - 9.9504 v_3 v_2 \sin(0 - \delta_2 + 1.4711)$$

Example

$$+\sqrt{(p_{ik}(\cdot))^2 + (q_{ik}(\cdot))^2} \leq s_{ik}^{\max}$$



Capacity limit per line

$$\Big[\left(9.9504\cos(-1.4711)v_1^2 - 9.9504v_1v_2\cos(\delta_1 - \delta_2 + 1.4711)\right)^2 + \left(-9.9504\sin(-1.4711)v_1^2 - 9.9504v_1v_2\sin(\delta_1 - \delta_2 + 1.4711)\right)^2\Big]^{1/2} \leq 0.25$$

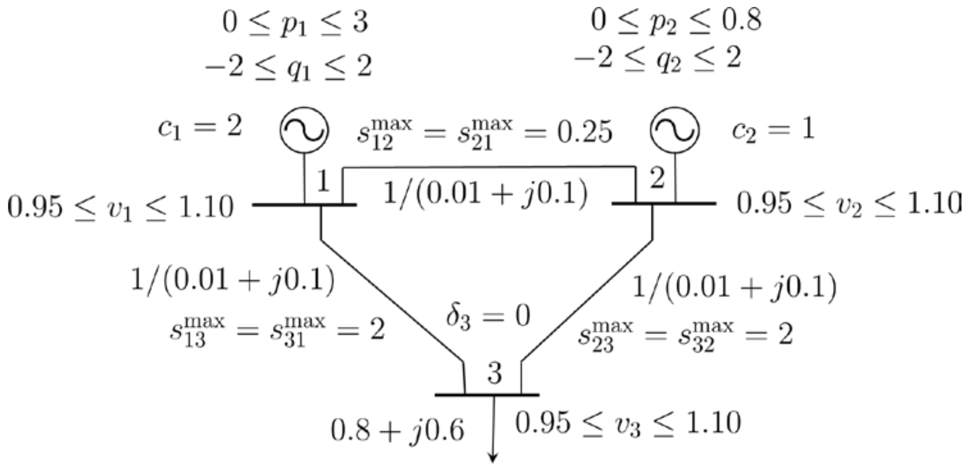
$$\Big[\left(9.9504\cos(-1.4711)v_1^2 - 9.9504v_1v_3\cos(\delta_1 - 0 + 1.4711)\right)^2 + \left(-9.9504\sin(-1.4711)v_1^2 - 9.9504v_1v_3\sin(\delta_1 - 0 + 1.4711)\right)^2\Big]^{1/2} \leq 2$$

$$\Big[\left(9.9504\cos(-1.4711)v_2^2 - 9.9504v_2v_3\cos(\delta_2 - 0 + 1.4711)\right)^2 + \left(-9.9504\sin(-1.4711)v_2^2 - 9.9504v_2v_3\sin(\delta_2 - 0 + 1.4711)\right)^2\Big]^{1/2} \leq 2$$

Example

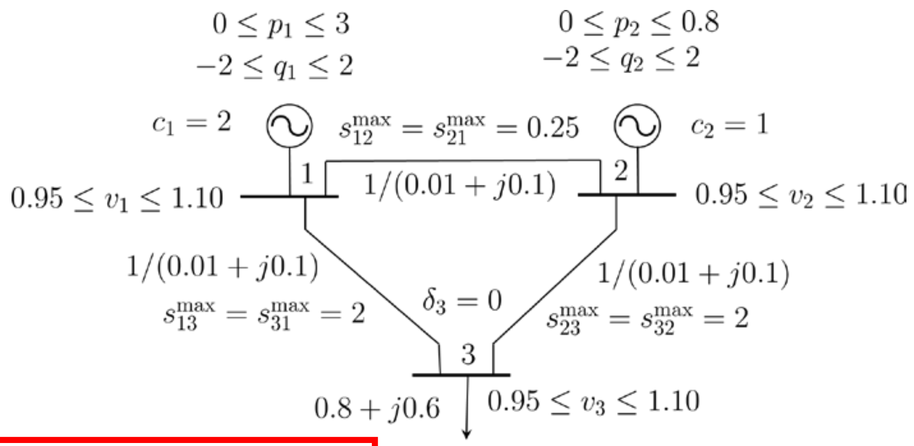
Other limits

$$0.95 \le v_1 \le 1.10, \quad 0.95 \le v_2 \le 1.10, \quad 0.95 \le v_3 \le 1.10$$
$$0 \le p_1 \le 3, \quad 0 \le p_2 \le 0.8$$
$$-2 \le q_1 \le 2, \quad -2 \le q_2 \le 2$$
$$-\pi \le \delta_1 \le \pi, \quad -\pi \le \delta_2 \le \pi$$



Just the math

Example

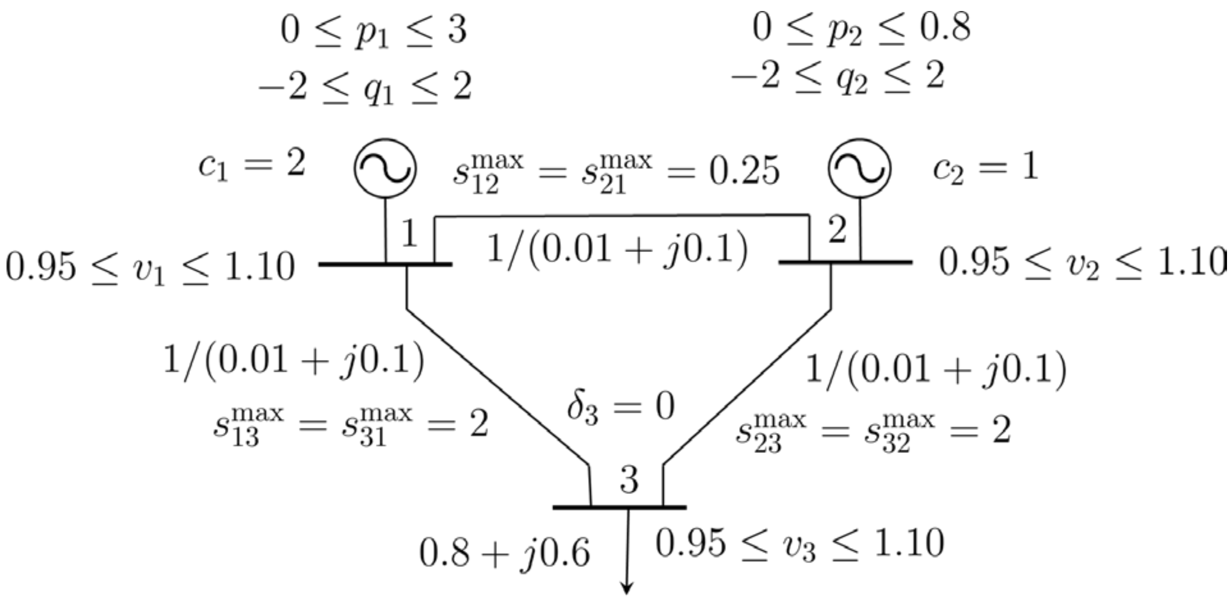


$$\min_{p_1,p_2,q_1,q_2,v_1,v_2,v_3,\delta_1,\delta_2} \quad 2p_1 + 1p_2$$

s.t.

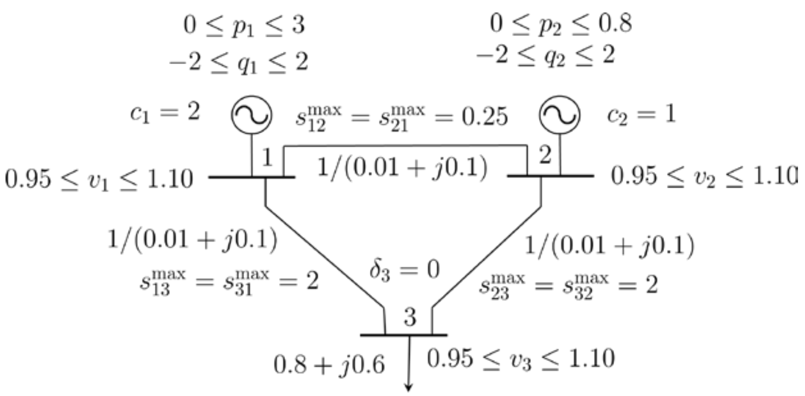
$$p_1 = 9.9504\cos(-1.4711)v_1^2 - 9.9504v_1v_2\cos(\delta_1 - \delta_2 + 1.4711) +$$
$$9.9504\cos(-1.4711)v_1^2 - 9.9504v_1v_3\cos(\delta_1 - 0 + 1.4711)$$
$$p_2 = 9.9504\cos(-1.4711)v_2^2 - 9.9504v_2v_1\cos(\delta_2 - \delta_1 + 1.4711) +$$
$$9.9504\cos(-1.4711)v_2^2 - 9.9504v_2v_3\cos(\delta_2 - 0 + 1.4711)$$
$$-0.8 = 9.9504\cos(-1.4711)v_3^2 - 9.9504v_3v_1\cos(0 - \delta_1 + 1.4711) +$$
$$9.9504\cos(-1.4711)v_3^2 - 9.9504v_3v_2\cos(0 - \delta_2 + 1.4711)$$

Example



$$
\begin{aligned}
q_1 &= -9.9504 \sin(-1.4711) v_1^2 - 9.9504 v_1 v_2 \sin(\delta_1 - \delta_2 + 1.4711) - \\
&\quad 9.9504 \sin(-1.4711) v_1^2 - 9.9504 v_1 v_3 \sin(\delta_1 - 0 + 1.4711) \\
q_2 &= -9.9504 \sin(-1.4711) v_2^2 - 9.9504 v_2 v_1 \sin(\delta_2 - \delta_1 + 1.4711) - \\
&\quad 9.9504 \sin(-1.4711) v_2^2 - 9.9504 v_2 v_3 \sin(\delta_2 - 0 + 1.4711) \\
-0.6 &= -9.9504 \sin(-1.4711) v_3^2 - 9.9504 v_3 v_1 \sin(0 - \delta_1 + 1.4711) - \\
&\quad 9.9504 \sin(-1.4711) v_3^2 - 9.9504 v_3 v_2 \sin(0 - \delta_2 + 1.4711)
\end{aligned}
$$

Example

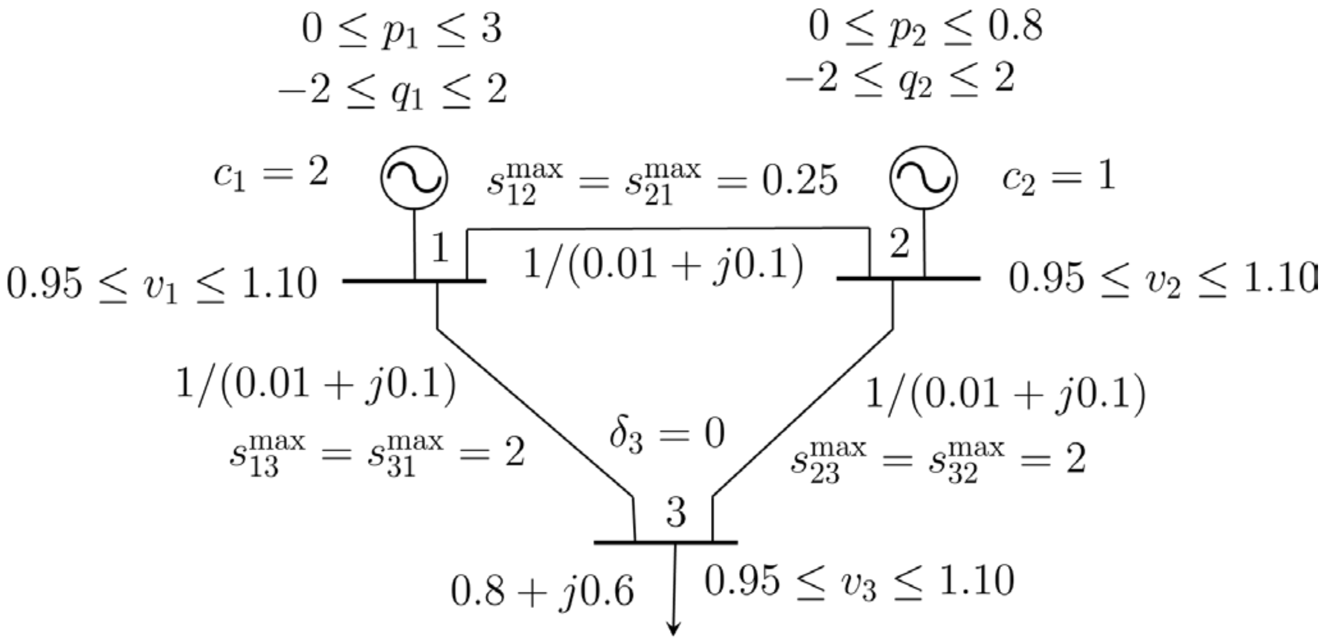


$$\left[\left(9.9504\cos(-1.4711)v_1^2 - 9.9504v_1v_2\cos(\delta_1 - \delta_2 + 1.4711)\right)^2 + \left(-9.9504\sin(-1.4711)v_1^2 - 9.9504v_1v_2\sin(\delta_1 - \delta_2 + 1.4711)\right)^2\right]^{1/2} \le 0.25$$

$$\left[\left(9.9504\cos(-1.4711)v_1^2 - 9.9504v_1v_3\cos(\delta_1 - 0 + 1.4711)\right)^2 + \left(-9.9504\sin(-1.4711)v_1^2 - 9.9504v_1v_3\sin(\delta_1 - 0 + 1.4711)\right)^2\right]^{1/2} \le 2$$

$$\left[\left(9.9504\cos(-1.4711)v_2^2 - 9.9504v_2v_3\cos(\delta_2 - 0 + 1.4711)\right)^2 + \left(-9.9504\sin(-1.4711)v_2^2 - 9.9504v_2v_3\sin(\delta_2 - 0 + 1.4711)\right)^2\right]^{1/2} \le 2$$

Example



$$0.95 \leq v_1 \leq 1.10, \quad 0.95 \leq v_2 \leq 1.10, \quad 0.95 \leq v_3 \leq 1.10$$

$$0 \leq p_1 \leq 3, \quad 0 \leq p_2 \leq 0.8$$

$$-2 \leq q_1 \leq 2, \quad -2 \leq q_2 \leq 2$$

$$-\pi \leq \delta_1 \leq \pi, \quad -\pi \leq \delta_2 \leq \pi$$

Example

The solution in per unit (angles in radians) is

$$p_1^* = 0.03090686, \quad p_2^* = 0.77426500$$
$$q_1^* = 0.32091673, \quad q_2^* = 0.33080186$$
$$v_1^* = 1.09770030, \quad v_2^* = 1.10000000, \quad v_3^* = 1.06635924$$
$$\delta_1^* = 0.02123196, \quad \delta_2^* = 0.04191265$$

GAMS code

GAMS code

A simple input GAMS file to solve this problem is provided below below.

GAMS code: simple

```
parameters
yl /  9.95037190209989/
al / -1.47112767430373/;
variables
z, p1, p2, q1, q2, v1, v2, v3, d1, d2;
p1.lo = 0; p2.lo = 0;
p1.up = 3; p2.up = 0.8;
q1.lo = -2; q2.lo = -2;
q1.up =  2; q2.up =  2;
v1.lo = .95; v2.lo = .95; v3.lo = .95;
v1.up = 1.1; v2.up = 1.1; v3.up = 1.1;
d1.lo = -pi; d2.lo = -pi;
d1.up =  pi; d2.up =  pi;
v1.l=1; v2.l=1; v3.l=1;
d1.l=0; d2.l=0;
```

GAMS code: simple

```
equations
of, bp1, bp2, bp3, bq1, bq2, bq3, l12, l21, l13, l31, l23, l32;
of.. z =e= 2*p1 + p2;
bp1.. p1   =e= yl*cos(al)*(v1)**2-yl*v1*v2*cos(d1-d2-al)+
               yl*cos(al)*(v1)**2-yl*v1*v3*cos(d1- 0-al);
bp2.. p2   =e= yl*cos(al)*(v2)**2-yl*v2*v1*cos(d2-d1-al)+
               yl*cos(al)*(v2)**2-yl*v2*v3*cos(d2- 0-al);
bp3.. -0.8 =e= yl*cos(al)*(v3)**2-yl*v3*v1*cos( 0-d1-al)+
               yl*cos(al)*(v3)**2-yl*v3*v2*cos( 0-d2-al);
bq1.. q1   =e= -yl*sin(al)*(v1)**2-yl*v1*v2*sin(d1-d2-al)-
                yl*sin(al)*(v1)**2-yl*v1*v3*sin(d1- 0-al);
bq2.. q2   =e= -yl*sin(al)*(v2)**2-yl*v2*v1*sin(d2-d1-al)-
                yl*sin(al)*(v2)**2-yl*v2*v3*sin(d2- 0-al);
bq3.. -0.6 =e= -yl*sin(al)*(v3)**2-yl*v3*v1*sin( 0-d1-al)-
                yl*sin(al)*(v3)**2-yl*v3*v2*sin( 0-d2-al);
```

GAMS code: simple

```
l12.. sqrt(sqr(yl*cos(al)*(v1)**2-yl*v1*v2*cos(d1-d2-al))+
           sqr(-yl*sin(al)*(v1)**2-yl*v1*v2*sin(d1-d2-al))) =l= 0.25;
l21.. sqrt(sqr(yl*cos(al)*(v2)**2-yl*v2*v1*cos(d2-d1-al))+
           sqr(-yl*sin(al)*(v2)**2-yl*v2*v1*sin(d2-d1-al))) =l= 0.25;
l13.. sqrt(sqr(yl*cos(al)*(v1)**2-yl*v1*v3*cos(d1- 0-al))+
           sqr(-yl*sin(al)*(v1)**2-yl*v1*v3*sin(d1- 0-al))) =l= 2;
l31.. sqrt(sqr(yl*cos(al)*(v3)**2-yl*v3*v1*cos( 0-d1-al))+
           sqr(-yl*sin(al)*(v3)**2-yl*v3*v1*sin( 0-d1-al))) =l= 2;
l23.. sqrt(sqr(yl*cos(al)*(v2)**2-yl*v2*v3*cos(d2- 0-al))+
           sqr(-yl*sin(al)*(v2)**2-yl*v2*v3*sin(d2- 0-al))) =l= 2;
l32.. sqrt(sqr(yl*cos(al)*(v3)**2-yl*v3*v2*cos( 0-d2-al))+
           sqr(-yl*sin(al)*(v3)**2-yl*v3*v2*sin( 0-d2-al))) =l= 2;
```

GAMS code: simple

```
model simpleopf /all/;
solve simpleopf using nlp minimizing z;
option decimals = 8;
display p1.l, p2.l, q1.l, q2.l, v1.l,
v2.l, v3.l, d1.l, d2.l;
```

GAMS code: simple

```
----      46 VARIABLE p1.L                 =    0.03090686
             VARIABLE p2.L                 =    0.77426500
             VARIABLE q1.L                 =    0.32091674
             VARIABLE q2.L                 =    0.33080185
             VARIABLE v1.L                 =    1.09770030
             VARIABLE v2.L                 =    1.10000000
             VARIABLE v3.L                 =    1.06635924
             VARIABLE d1.L                 =    0.02123196
             VARIABLE d2.L                 =    0.04191265
```

This is it!