Higher-Order SPT phases

What is an SPT?

Intuitively, an SPT has a "trivial" bulk and nontrivial boundary

"Trivial" bulk: gapped

short-range entangled (i.e. no topological order)

non-degenerate (even on torus)

Trivial phase is a product state

Ex: insulator, paramagnet

"Nontrivial" boundary: gapless (SSB or not)

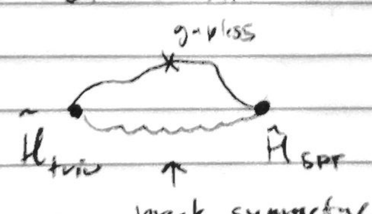
degenerate

topologically ordered

Realize symmetries nonlocally, not onsite, non-linearly

In reality: 't Hooft anomaly

Phase of matter:



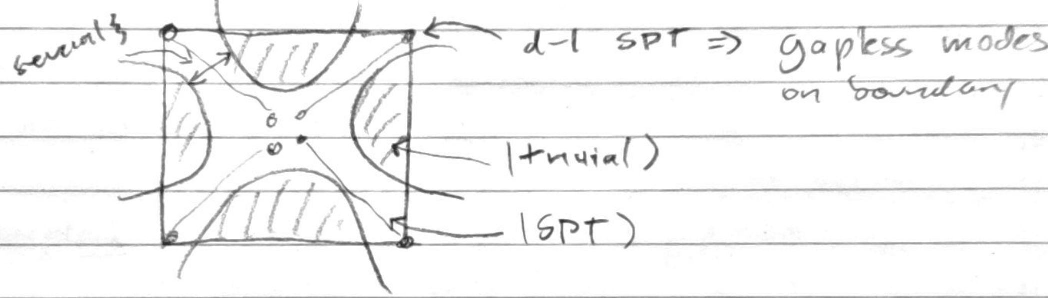
HOSPTs, ctd

Disentangling operation:

Can use a local, symmetry (G_0) preserving unitary operation (finite-depth quantum circuit) to "disentangle"

-> cannot happen everywhere or not |SPT⟩

Now, include crystal symmetry as well:



The disentanglers must act in a crystal-symmetric way, so the operator acts in several places

BUT: cannot cover the whole system (by assumption), so remainder must be SPT

Some technical points:

Need G_c to act freely on boundary

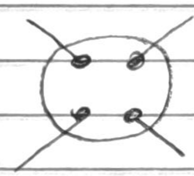
G_c and G_0 must commute

Lower-dim SPT's live on domain walls of the crystal symmetry

HOSPT, ctd

What happens in bulk?

By assumption, have several G_0 d-1 SPT's fusing together at rotation center



Ends must fuse to a trivial, gapped state to be an SPT bulk

Example: $C_n \times SO(3)$ in 2+1d

Same picture as before

1+1d systems are Haldane chain SO(3) SPT's

For $n=4$ (or even), have in bulk:

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \underbrace{2 \oplus \cdots \oplus 0}_{\text{linear rep, gapped by assumption of paramagnet}}$$

For $n=3$ (or odd)

$$= \overbrace{\frac{3}{2} \oplus \cdots \oplus \frac{1}{2}}^{\text{not linear rep = bad}}$$

Overall classification is $\mathbb{Z}_2$ since Haldane is $\mathbb{Z}_2$

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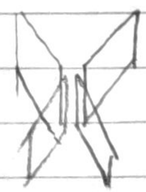
In higher dimension, can have hinges or other hypersurfaces



for C_4 example

Here, fuse edges (often to non-chiral)

Cohomology

Thought that "all" boson SPTs fit into

$$H^{d+1}(G_c \times G_0, U(1))$$

Decomposing gives, for example,

$$H^1(C_4, H^2(SO(3), U(1)))$$

which is exactly captured by above.