Example Problem: Interference Part 2
Physics 1251
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1. **Two speakers are coherent sources of sound that emit sound waves in phase. Point P is a distance $d_1 = 2.45\,\text{m}$ from one speaker and a distance $d_2 = 1.78\,\text{m}$ from the other speaker. The speed of sound is $343\,\text{m/s}$. Find three different frequencies that will experience *destructive* interference at P.**

   For destructive destructive interference at P, we want the path length difference between each of the speakers and P to be equal to some odd half-integer multiple of wavelength λ.

   $$\Delta d = d_2 - d_1 = (m + 1/2)\lambda;\ m = 0, 1, 2, ...$$

   Additionally, we know that $v = f\lambda$, so

   $$\Delta d = \frac{(m + 1/2)v}{f}$$

   Solving for f, we find:

   $$(m = 0)\ \boxed{f_0 = \frac{v}{2\Delta d} = 256\,\text{Hz}}\quad (m = 1)\ \boxed{f_1 = \frac{3v}{2\Delta d} = 768\,\text{Hz}}\quad (m = 2)\ \boxed{f_2 = \frac{5v}{2\Delta d} = 1280\,\text{Hz}}$$

2. **Light of wavelength $\lambda = 600\,\text{nm}$ passes through a pair of slits; each slit is $50\,\mu\text{m}$ wide and the slits are separated by $170\,\mu\text{m}$.**

   (a) **How many bright fringes occur on a screen $1.3\,\text{m}$ wide that is $1.5\,\text{m}$ past the slits?**

   We want to count the total number of bright fringes in the double slit pattern out to the angle θ, where the end of the screen is. The screen is 1.5 m away from the slits and the edge of the screen is located at a distance of $1.3/2 = 0.65\,\text{m}$ from its center, so

   $$\theta = \arctan\frac{0.65}{1.5} = 23.4°$$

   Next we can use the equation representing double slit interference maxima to calculate the order m of the last bright fringe on the screen:

   $$d\sin\theta = m\lambda \implies m = \frac{d\sin\theta}{\lambda} \implies m = 112.7$$

   This means there are 112 fringes to the right, and to the left, of the central bright maximum at $m = 0$. So in total, we have:

   $$N = 112 + 112 + 1 = \boxed{225\,\text{fringes}}$$

   (b) **How many bright fringes occur in the whole pattern?**

   The last bright fringe in the entire pattern occurs when the angle of interference is $\theta = 90°$. Thus, we set $\sin\theta = 1$ in the double slit interference maximum equation and find the order of the last interference maximum:

   $$d \cdot 1 = m\lambda \implies m = \frac{170\mu\text{m}}{600\mu m} = 283.3$$

   Meaning there are 283 full fringes on each side of the diffraction pattern, and then the one in the middle. So the total in the pattern is

   $$N = 283 + 283 + 1 = \boxed{567\,\text{fringes}}$$

3. **Two colors of light pass through a double slit at the same time. The second interference minimum of color A is at the same angle as the third maximum of color B. What is the ratio of λ_A/λ_B?**

   The equations describing interference maxima and minima for a double slit are:

$$d\sin\theta = m\lambda \text{ (maxima)}$$
$$d\sin\theta = (m - 1/2)\lambda \text{ (minima)}$$

   Since the second minimum of color A and the third maxima of color B are located at the same angle, we can write:

$$d\sin\theta = \frac{3}{2}\lambda_A \;\; \& \;\; d\sin\theta = 3\lambda_B$$
$$\frac{3}{2}\lambda_A = 3\lambda_B \Longrightarrow \boxed{\frac{\lambda_A}{\lambda_B} = 2}$$

4. **Two colors of light pass through a single slit at the same time. The second diffraction minimum of color A is at the same angle as the third maximum of color B. What is the ratio of λ_A/λ_B?**

   The equations describing interference maxima and minima for a single slit are:

$$d\sin\theta = m\lambda \text{ (minima)}$$
$$d\sin\theta = (m + 1/2)\lambda \text{ (maxima)}$$

   Again, the second minimum of color A is at the same angle as the third maximum of color B:

$$d\sin\theta = \frac{7}{2}\lambda_B \;\; \& \;\; d\sin\theta = 2\lambda_A$$
$$\Longrightarrow \frac{7}{2}\lambda_A = 2\lambda_B \Longrightarrow \boxed{\frac{\lambda_A}{\lambda_B} = \frac{7}{4} = 1.75}$$

5. **A** 0.500 **micrometers thick film of $n = 1.25$ soap is suspended in mid-air.**

   (a) **For what wavelengths of visible light is the reflected light a maximum? For what wavelengths of visible light is the reflected light a minimum?**

   We are going to employ the equations for constructive and destructive thin film interference in the case where n_3 (air) $< n_2$ (soap) $> n_1$ (air). Then we have:

$$2t = (m + 1/2)\frac{\lambda}{n} \text{ (maxima)} \;\; \& \;\; 2t = m\frac{\lambda}{n} \text{ (minima)}$$

   For maxima, the only m with a λ in the visible spectrum $(400 - 700\,\text{nm})$ is $m = 2 \Longrightarrow$ $\boxed{\lambda_2 = 500\,\text{nm}}$. For minima, $m = 2$ and $m = 3$ both work with $m = 2 \Longrightarrow \boxed{\lambda_2 = 625\,\text{nm}}$ and $m = 3 \Longrightarrow \boxed{\lambda_3 = 416\,\text{nm}}$.

   (b) **If instead of being suspended in mid-air, the soap film was applied to a piece of glass (with, say, $n_{glass} = 1.5$), what would those wavelengths be?**

   If the film as applied to glass, then now, n_3 (glass) $> n_2$ (soap) $> n_1$ (air), so the conditions for destructive and constructive interference change, and our answers above just swap roles. Maxima are now found at 625 nm and 416 nm, and minima at 500 nm.