Exam 1 Show Work Practice Problems
Physics 1251
TA: Brian Clark

Disclaimer: Brian has ZERO intel about what will be covered on the midterm. This is a collection of problems that will force you to review concepts that were covered in lecture, homework, and quizzes. My instincts tell me the questions I have provided here are, on net, slightly more difficult than you will encounter on the actual exam, but again, no promises.

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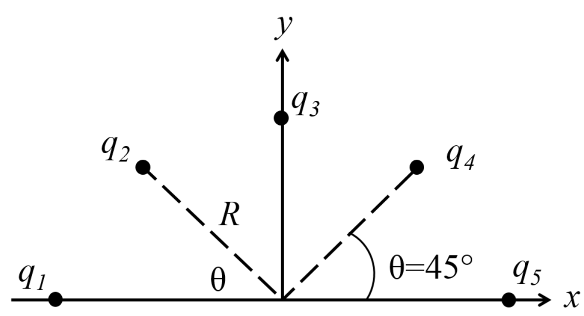
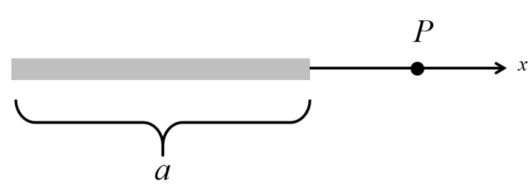
1. Five point charges are uniformly distributed in a semicircle as shown in the figure. Each charge is fixed in place a distance R from the origin. Note that $q_1 = q_5$ and $q_2 = q_4$.



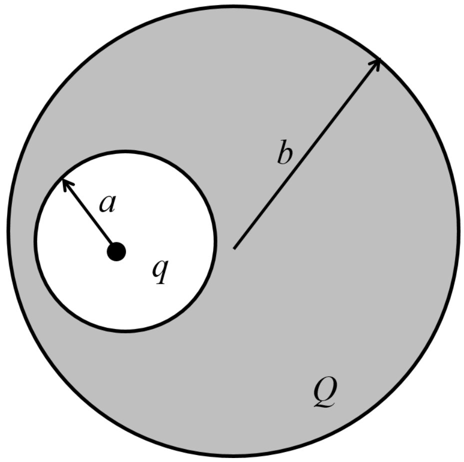
   (a) Calculate the **net** electric fields (**magnitude and direction**) at the origin due to the five charges. Report your answer in terms of q_1, q_2, q_3, R, and fundamental constants of nature like k and ϵ_0.
   (b) A charge $+Q$ is now placed at the origin. Determine the the **net** electric force (**magnitude and direction**) that the charge $+Q$ experiences. If you found the answer to part (a), you can report your answer in terms of that field.
   (c) If $q_1 = q_5 = +10\,\mu\,\mathrm{C}$ and $q_2 = q_4 = +20\,\mu\,\mathrm{C}$, determine the charge of q_3 if $+Q$ experiences no net force when placed at the origin. Explain your reasoning, and show your work.

2. A total amount of positive charge Q is distributed uniformly along a thin rod of length a. We are intersted in the electric field and potential at point P, which lies along the same line as the rod and a distance r away from the closer end of the rod.



   (a) Find the **net** electric fields (**magnitude and direction**) at point P.
   (b) Find the potential at point P relative to infinity. That is, you may take $V = 0$ to be infinitely far away, as usual.
   (c) You should be able to find the electric field from part (a) using the potential you calculated in part (b). Show that if you do this, you get the same answer as in part (a).

3. A conducting sphere of radius b has an off-center spherical cavity of radius a carved out of its interior. A total amount of charge Q is put on the conductor, and a point charge q is placed at the center of the spherical hole. Charges in the conductor have adjusted to this setup and are no longer moving. The picture shows a cross section of the sphere.



(a) How much charge is located on the inner surface of the conductor? Be sure to explain your reasoning, and draw the Gaussian surface of choice, if you choose to use Gauss's law.

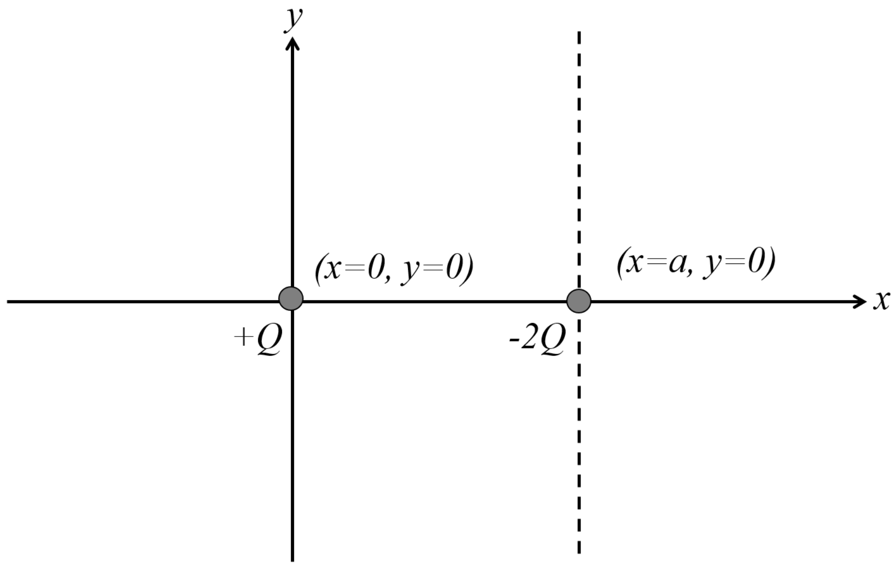
(b) What is the total amount of charge located on the outer surface of the conductor? Be sure to explain your reasoning, and draw the Gaussian surface of choice, if you choose to use Gauss's law.

(c) A small charge q' is placed outside the conductor, a distance r away from the conductors center ($r > B$). Find the magnitude of the force on q'. Assume that q' is so small that it doesn't cause any significant redistribution of charges on the conductor, i.e., the locations of those charges doesn't change from part (b).

4. A positive charge Q, is fixed at the origin $(x = 0, y = 0)$, and a charge $-2Q$ is fixed at the point $(x = a, y = 0)$, with $a > 0$.

   (a) Consider points on the x axis that are very far away from the origin, either for positive or for negative x. If a is negligible in comparison to the magnitude of x, what is the electric potential at x, relative to zero at infinity?

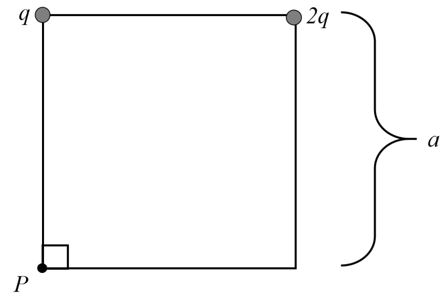
   (b) Using the diagram below, plot a rough graph of the electric potential $V(x)$ as a function of x for all x. A good way to start may be to sketch the potential close to the two charges, where one charge will dominate.



   (c) At which points on the x-axis is the electric potential zero, relative to zero at infinity? There may be no such points, or one, or more than one; however many there are, be sure to locate them all.

   (d) A third charge, $-3Q$ is brought up from a very large distance to the point on the x-axis with $x = 2a$. If $Q = 2\,\mu\mathrm{C}$ and $a = 0.5\,\mathrm{m}$, how much external work, in Joules, will be needed to bring up this third charge? You may assume the charge is moved slowly, so it always had negligible kinetic energy.

5. Two point charges, q and $2q$ are held at fixed corners of one side of a square of side a, as shown.



(a) What are the x **and** y components of the electric fields at point P, in the lower left-hand corner of the square?

(b) What is the potential V at a point P, assuming $V = 0$ infinitely far away?

(c) A third point charge, $3q$, is held at point P. Using your results in (a) and/or b, find the components (in Newtons) of the non-electric force needed to hold this new charge $3q$ in position. Take $a = 1.0\,\text{cm}$ and $q = 1.0\,\text{nC}$. [If you could not answer (a), you make take $E_x = -q/(4\pi\epsilon_0 a^2)$, $E_y = -2q/(4\pi\epsilon_0 a^2)$. If you could not answer (b), you may take $V = 2q/(4\pi\epsilon_0 a)$.]

(d) The third charge, $3q$, is released from rest from point P and is pushed away by the electrical repulsion from the charges q and $2q$. Using your result from (a) and/or (b), find how much kinetic energy (in Joules) it will have when it gets very far away from the other charges. Take $a = 1.0\,\text{cm}$ and $q = 1.0\,\text{nC}$. [If you could not answer (a), you make take $E_x = -q/(4\pi\epsilon_0 a^2)$, $E_y = -2q/(4\pi\epsilon_0 a^2)$. If you could not answer (b), you may take $V = 2q/(4\pi\epsilon_0 a)$.]

6. Two charged solid spherical conductors of radius $R_1 = 6.0\,\text{cm}$ and $R_2 = 2.0\,\text{cm}$ are separated by a distance much greater than the radii of the spheres, and the two spheres are connected by a long, thin conducting wire. A total charge of $Q = -80.0\,\text{nC}$ is placed on the smaller sphere. You may assume there is no net charge on the wire when equilibrium is reached.

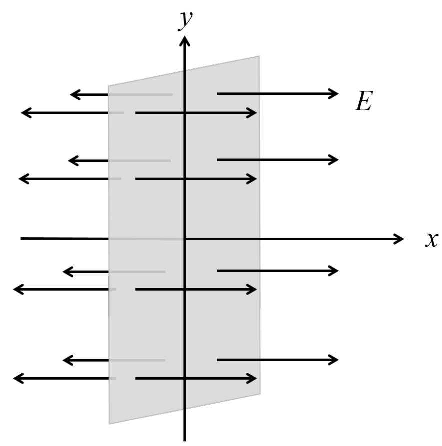
(a) Determine how much charge is on each sphere after equilibrium is reached, and the charges come to rest.

(b) Find the electric potential at the surfaces of the spheres assuming that $V = 0$ an infinite distance from the spheres.

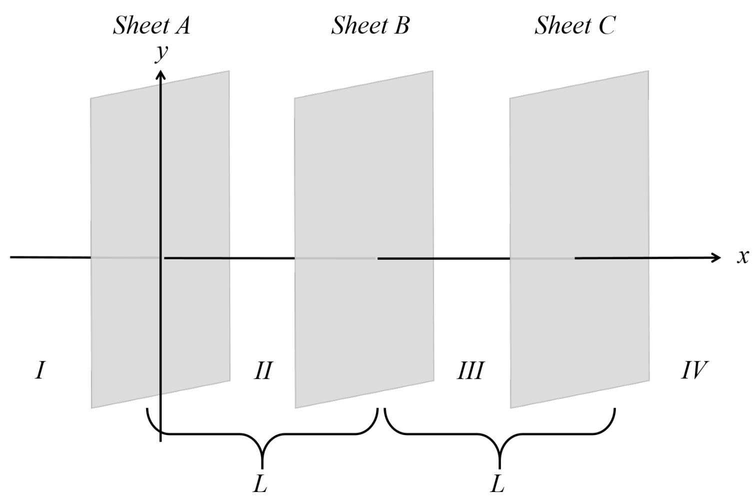
(c) Is the electric field at the surface of the smaller sphere greater than, less than, or equal to the electric field at the surface of the larger sphere? Explain your reasoning, and show your work.

7. The following questions pertain to infinite insulating sheets of charge with negligible thickness.

   (a) An infinite insulating sheet of positive charge with surface charge density σ is depicted in the figure below with representative electric field lines. Use Gauss's law to determine the magnitude of the electric field $\vec{E}$. Be sure to 1) draw your Gaussian surface on the figure below (or draw a new picture all together if you would like), 2)draw and label all area vectors on your Gaussian surface, and 3) carefully explain your reasoning when you compute the field.



   (b) Determine the net electric field (**magnitude and direction**) in regions I, II, III, IV due to the three evenly spaced infinite insulating charge sheets shown below. Note that sheets A and C have negative charge with a surface charge density $-\sigma$, and sheet B has positive surface charge density 2σ. The spacing between neighboring sheets is L. Clearly explain your reasoning and show your work.

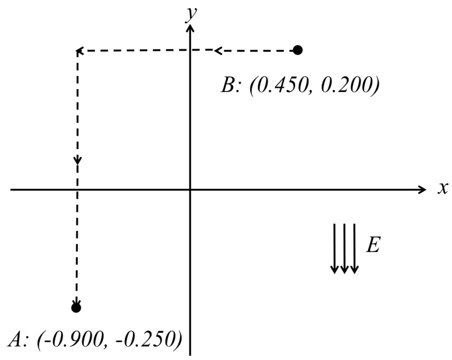


8. A charge $-Q$ is placed at the center of two concentric, conducting shells. The smaller shell has inner radius $r = a$ and outer radius $r = b$. A total charge of $+Q$ has been placed on this smaller conducting shell. The larger shell has inner radius $r = c$ and outer radius $r = d$. A total charge of $+2Q$ has been placed on this outer conducting shell.

   (a) Determine the charge on the conducting surfaces at $r = a$, $r = b$, $r = c$, $r = d$. Explain your reasoning.

   (b) Use Gauss's law to find the magnitude of the electric field everywhere in space. This means you should have a separate answer for $0 < r < a$, $a < r < b$, $b < r < c$, $c < r < d$, $r > d$.

9. A uniform electric field of magnitude $270\,\mathrm{V/M}$ is directed in the negative y direction as shown in the figure below. The coordinates of A are $(x = -0.900\,\mathrm{m}, y = -0.250\,\mathrm{m})$ and those of point B are $(x = 0.450\,\mathrm{m}, y = 0.200\,\mathrm{m})$. Calculate the eletric potential difference $V_A - V_B$ using the dashed line path.



10. A not unreasonable model of the Earth is one where the Earth's surface and the lower layer of clouds act as a parallel plate capacitor. So, regarding the Earth and a cloud layer $900\,\mathrm{m}$ above the Earth as "plates" of a capacitor, assume the cloud layer has an area of $1.00\,\mathrm{km}^2$ and the air between the cloud and the ground is pure and dry. Assume the charge builds up on the cloud and on the ground until a uniform electric field of $4.00 \times 10^6\,\mathrm{N/C}$ throughout the space between them makes the air break down and conduct electricity as a lightning bolt.

    (a) Calculate the capacitance of the Earth-cloud layer system.

    (b) What is the maximum charge a cloud can hold?

    As a side note, this model isn't very bad! When lightning strikes a plane or radio tower, it experimentally deposits something like $1.5\,\mathrm{C}$ *of charge.*