Cooperative Extremum Seeking Control via Sliding Mode for Distributed Optimization

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Abstract— **In this paper, a cooperative distributed optimization method via sliding mode extremum seeking (ES) control for a class of large-scale interconnected systems is presented. In this approach, a consensus algorithm is exploited to communicate the value of the global cost function to the ES controllers. Then, each sliding mode ES controller is designed such that a multivariable cost function is optimized in a cooperative fashion. The stability and convergence conditions for the ES controllers are determined and sufficient conditions for the distributed scheme to converge to the vicinity of the optimal points are driven. The application of the proposed scheme to a real-world example is investigated and simulations are provided to illustrate the theoretical results and demonstrate their potential use.**

I. INTRODUCTION

Conventional control methods address stabilization, regulation, and/or fixed set-point tracking for a given dynamic system. However, many applications demand optimal set points unknown in advance or that must vary over time, such as for Anti-lock Braking System control facing unpredictable changes in road conditions. Extremum seeking (ES) control is a form of optimal control that deals with a situation in which the cost function to be optimized is not exactly known, but can be measured. ES control is also a method of adaptive control in which the objective of the controller is to steer the system output to follow a non-predetermined optimal operating point [1].

Prior work, to a large extent, has analyzed and applied the sliding mode ES control in centralized fashion. Korovin and Utkin [2] introduce the use of sliding mode in self-optimization for static mapping. The main idea is to select a control law such that the system output tracks a monotonic decreasing (increasing) function in time towards the minimum (maximum). Furthermore, Ozguner and his coworker generalized the method in the presence of dynamics [3]–[5]. A more recent study extends the single variable sliding mode ES to a multivariable was reported in [6]. It was successfully applied to a variety of applications such as: Anti-lock Braking System (ABS) [7], source seeking [8], and Maximum Power Point Tracking (MPPT) [9].

Although centralized control architectures have proven to be effective for some applications, decentralized control and distributed process improve the scalability of systems. For this reason, the literature shows increasing interest in applying decentralized control and distributed optimization with a variety of gradient and sub-gradient methods [10]–[12]. However, when the gradient of the cost function is not available and the only measurement of the cost function is available, extremum seeking control appears to be the most promising approach to solve the problem. Recently, much consideration has been given to how to apply ES control to distributed systems. The current distributed ES control approaches can be divided into two major categories. A non-cooperative extremum seeking setting where the interaction between the controllers seen as an n-person noncooperative dynamic game and the goal is to find the Nash equilibrium solution using the extremum seeking control. Pan and Ozguner extend the results from the sliding mode ES control to non-cooperative distributed optimization by introducing n sliding surfaces for each player [13]. Similarly, Frihauf et al. [14] sought to solve this problem by extending the periodic perturbation approach [1]. The other category is the cooperative based ES control, where the objective is to optimize a global cost function via local controllers and reach a common objective. The researchers in [15], propose to utilize the primal-dual technique to reach a saddle point. Similar ideas are analyzed in [16]–[21], where the authors introduce a dynamic average consensus to share an estimate of the total cost function with each controller. Distinct from the previous studies, the author of [19] formulates the ES setup to solve a class of optimal resource allocation problems. To reach an optimal assignment of limited resources, consensus dynamics are used to provide each controller with an estimate of the available resources.

In this paper, a decentralized and cooperative sliding mode ES control is proposed. The main concept behind this approach is to introduce a consensus algorithm to communicate the value of the global cost function to the controllers. Next, the consensus algorithm is interfaced suitaby with a sliding mode ES control in order to find the optimal solution. Different from existing works, this paper has three distinguishing features: First, in the context of distributed and cooperative scheme, the existing work relies on the use of sinusoidal perturbation signals, while this paper employs a sliding mode ES control. Sliding mode ES control has the potential to simplify the implementation and improve the convergence rate. Second, ES via sliding mode was previously proposed to find Nash solution in a non-cooperative game [13]. Since Nash solution is not the socially optimal solution, this paper seeks to extend the result to solve the problem in a cooperative fashion. Third, the stability and the convergence analysis are proven under less stringent conditions by eliminating the weak coupling assumption

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between the decision variables.

The remaining work in this paper is developed as follows: The problem statements and assumptions are outlined in section II. Section III specifies the proposed controller design, followed by stability and convergence analysis in section IV. The application of the proposed schemes to optimize energy production in wind farms is investigated in section V, where successful application of distributed ES control is achieved. Simulations are provided to illustrate the theoretical results and demonstrate its potential use.

II. Preliminaries and Problem Formulation

Throughout this paper a network of n controllers, will be refereed to as *agents*, is considered. Each agent i for $i = 1,2,..,n$ is able to measure its local cost function $J_i(\theta(t))$, which depends on its own action, denoted by $\theta_i(t)$, as well as actions taken by its neighbors. Also, agents have the capability to exchange information over undirected graph $G = (V,E)$, with the vertex set $V = \{1,2,...,n\}$ and edge set $E \subset V \times V$.
Then, using the measurement of $J_i(\theta(t))$, the Multi-Agent System (MAS) objective is to perform a real-time optimization in a distributed and cooperative manner, and optimize a global cost function given by the sum of individual agent objectives

$$J(\theta(t)) = \sum_{i=1}^{n} J_i(\theta(t)), \tag{1}$$

where $J(\cdot) : \mathbb{R}^n \to \mathbb{R}$ is the global cost function. In this work, without loss of generality, the maximization case is considered:

$$\max_{\theta \in \mathbb{R}^n} \quad J(\theta(t)). \tag{2}$$

To proceed with the design of the cooperative ES control, the following assumptions are required to facilitate the analysis of the proposed solution.

Assumption 1: It is assumed that, the local cost functions $J_i(\theta(t))$ are concave functions i.e.

$$(\theta_i(t) - \theta_i^\star)\frac{\partial J(\theta(t))}{\partial \theta_i} < 0, \ \forall \theta_i(t) \neq \theta_i^\star. \tag{3}$$

and the solution to the problem (2) exists, unique and finite.

Assumption 2: For each agent i, the local cost function $J_i(\cdot), \ \ i \in V$ is smooth and continuously differentiable.

Assumption 3: For each θ_i, $i \in V$, the partial derivative of the total cost function function is bounded, i.e.

$$|\frac{\partial J}{\partial \theta_i}| < M_i, M_i \in \mathbb{R}$$

Assumption 4: The graph $G = (V,E)$ is fixed, balanced, and strongly connected.
Assumptions 1,2 and 3, will be needed for the stability and convergence properties of the sliding mode ES controller. Assumption 4 is introduced for the stability of the proposed consensus dynamic.

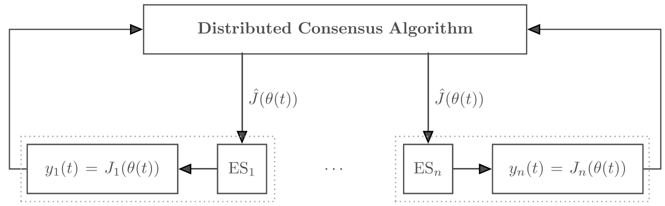

Fig. 1. The proposed scheme for the cooperative extremum seeking control. A *distributed* consensus algorithm receives measurement from each agent then share information about the global cost function.

III. Cooperative Sliding Mode ES Control

In order to solve the problem in a cooperative manner, each agent should have an estimate of the global cost (1). The idea is to introduce a consensus algorithm that communicates information about the mean of the global cost function in a distributed manner. Then, each controller will solve in real time a multivariable ES control problem. Fig. 1 demonstrates the proposed coordination scheme. As can be seen in the figure, each agent receives an estimate from the distributed consensus algorithm. Then, the ES controller uses the estimate to update the consensus with new value as a new decision has been taken by the local ES controller.

In the next section, the consensus dynamic used in this paper is elaborated on, then the design of the sliding mode extremum seeking controller is described.

A. Consensus Dynamic

In this section, we overview the consensus algorithm used in this work. In particular, similar to that of [18], we let each agent measure the mean of the total cost function, i.e.

$$\frac{1}{n}\sum_{i=1}^{n} J_i(\theta(t))$$

This can be achieved by using a dynamic consensus algorithm. In this work, the dynamic average consensus algorithm proposed by Freeman et al. [22] is considered. We use *dynamic* average consensus algorithm to make sure that the inputs continually drive the consensus algorithm in real time and to be able to track the average of the changing inputs. To elaborate more about the used consensus algorithm, we state some useful definitions:

Definition 1: An adjacency matrix A of the graph G satisfies $(i,j) \in E \implies a_{i,j} = 1$. Meaning the agent i can communicate with agent j. Since we assume un-directed graph we have $a_{i,j} = a_{j,i}$ ans $A = A^T$.

Definition 2: Let $D \in \mathbb{R}^{n \times n}$ be a diagonal matrix which elements d_{ii} equals the number of agents that agent i can commuincate with, the **Laplacian** matrix is given by $L = D - A$.
The consensus algorithm applies a distributed dynamic average consensus which is given by:

$$\begin{aligned}\dot{\hat{J}}_i(t) = &-\gamma \hat{J}_i(t) - \sum_{j \neq i} a_{ij}[\hat{J}_i(t) - \hat{J}_j(t)] \\ &+ \sum_{j \neq i} b_{ij}[w_i(t) - w_j(t)] + \gamma u_i(t)\end{aligned} \tag{4}$$

$$\dot{w}_i(t) = -\sum_{j\neq i} b_{ij}[\hat{J}_i(t) - \hat{J}_j(t)], \quad (5)$$

where $\hat{J}_i(t)$ is the estimated cost function for agent i, $w_i(t)$ is an auxiliary state, $u_i(t) \in \mathbb{R}$ is the dynamic input, and $\gamma > 0$ is a global parameter. The dynamic can be rewritten in compact form as:

$$\begin{bmatrix} \dot{\hat{J}} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -\gamma I - L_I & L_P \\ -L_P & 0 \end{bmatrix} \begin{bmatrix} \hat{J} \\ w \end{bmatrix} + \begin{bmatrix} \gamma I \\ 0 \end{bmatrix} u(t), \quad (6)$$

where $\hat{J}(t) = [\hat{J}_1(t) \;\; \hat{J}_2(t) \;\; \ldots \;\; \hat{J}_n(t)]^T$, and L_I is the integral Laplacian, constructed form the weights a_{ij}, and L_P is the proportional Laplacian, constructed form the weights b_{ij}. For this approach, $J_i(\theta(t))$ is chosen to be the input to the dynamic $u_i(t)$.

B. Sliding Mode ES Control

Fig. 2 shows a basic sliding mode based Extremum Seeking control scheme for general nonlinear system. The main idea of the sliding mode ES is to select a control law such that the system output tracks a monotonic increasing (decreasing) function in time towards the maximum (minimum). To design an ES controller with sliding mode, a switching function for the i_{th} agent is defined as:

$$s_i(t) = J_i(t) - g_i(t), \quad (7)$$

where $g_i(t)$ is a reference signal satisfies:

$$\dot{g}_i(t) = \rho, \; g_i(0) = 0, \quad \rho > 0. \quad (8)$$

That is, $g_i(t)$ is a increasing function of time. Next, let the variable structure control law be

$$v_i(t) = -k \; \mathrm{sgn}\left(\sin\left(\frac{\pi s_i(t)}{\alpha_i}\right)\right), \quad (9)$$

where α_i and k are positive design parameters. Also, let the decision variable θ_i be defined as:

$$\dot{\theta}_i(t) = v(t). \quad (10)$$

Finally, we made the following assumptions about the controller.

Assumption 5: The consensus system (6) is much faster than the one of the θ_i's dynamics, that is,

$$|\frac{d}{dt}\hat{J}(t)| >> |\frac{d}{dt}\theta(t)|. \quad (11)$$

This assumption ensures a time-scale separation between the consensus dynamic and the ES controller. This assumption is reasonable and can be satisfied by choosing a small k for each ES controller.

IV. Stability and Convergence Analysis

This section discusses the stability and the convergence of the proposed scheme. The stability of the consensus algorithm is discussed first. Then, the discussion will focus on our main result which is the stability and convergence analysis of the sliding mode ES controller. Namely, the analysis will be focused on the existence of sliding modes. The main theorem for a system of n variable will be stated and proved first. After that, we discuss an example of $n = 1$ for the clarity of the analysis.

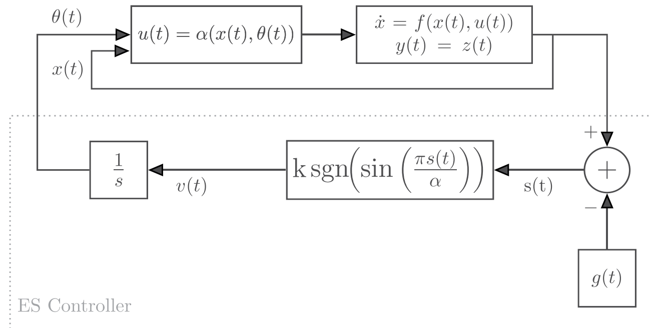

Fig. 2. Block diagram for Extremum Seeking control via sliding mode for general nonlinear system.

A. Convergence of the Consensus Algorithm

Lemma 1: Consider the consensus system (6) and let assumption 4 be satisfied. Then for any initial conditions the system states $[\hat{J}(t) \;\; w(t)]^T$ remain bounded and each state $\hat{J}_i(t)$ converges exponentially to $\frac{1}{n}\sum_{i=1}^n u_i(t)$ as $t \to +\infty$.

Proof: The proof can be found in [22]. ■

Using this lemma and under assumption 5, we have

$$J_i(t), J_j(t) \to \frac{1}{n}\sum_{i=1}^n J_i(\theta(t)) \quad \forall i,j \in \{1,2,...,n\}.$$

Therefore, throughout this section of analyzing the slow dynamics, $\hat{J}_i(\theta(t)) = \frac{1}{n}\sum_{i=1}^n J_i(\theta(t))$ is considered as the measured cost function for each agent i.

B. Existence of Sliding Mode

The condition for a sliding mode to exists is that the deviation from a switching surface $s(t)$ and its derivative $\dot{s}(t)$ should have opposite sign in the vicinity of the switching surface $s(t)$ [23]. For this paper, that is equivalent to saying that

$$\lim_{s_i(t)\to n\alpha^-} \dot{s}_i(t) > 0, \text{ and } \lim_{s_i(t)\to n\alpha^+} \dot{s}_i(t) < 0. \quad (12)$$

Theorem 1: For each agent i, with the switching function (7) and ES control input (9), let assumption 1 to assumption 5 hold. Then, the sliding mode existence condition is ensured and kept on manifold given by $s_i(t) = n\alpha, n \in \mathbb{Z}$ if the following conditions hold:

- For the given cost function (22), the sum of the absolute values of the partial derivative with respect to θ_i is grater than $\frac{\rho}{k}$, i.e.,

$$\sum_{i=1}^n \left| \frac{\partial J(\theta(t))}{\partial \theta_i} \right| > \frac{\rho}{k}, \; \forall i \in \{1,2,...,n\}. \quad (13)$$

- The ES controller parameters α_i is chosen such that:

$$\alpha_i \neq \alpha_j \;\text{ and }\; \alpha_i = 2^{i-1}\alpha, \alpha \in \mathbb{R}^+, \; \forall i,j \in \{1,2,...,n\} \quad (14)$$

Proof: Under lemma 1 we have the switching function for each agent i is given by:

$$s_i(t) = \hat{J}_i(\theta_1(t), \theta_2(t), ..., \theta_n(t)) - g(t).$$

and the time derivative is given by:

$$\begin{aligned}\dot{s}_i(t) &= \frac{\partial \hat{J}_i(\theta(t))}{\partial \theta_1}\dot{\theta}_1(t) + \ldots + \frac{\partial \hat{J}_i(\theta(t))}{\partial \theta_n}\dot{\theta}_n(t) - \rho \\ &= \sum_{j=1}^{n} \frac{\partial \hat{J}_i(\theta(t))}{\partial \theta_j}\dot{\theta}_j(t) - \rho \\ &= -\sum_{j=1}^{n} \frac{\partial \hat{J}_i(\theta(t))}{\partial \theta_j} k \ \mathrm{sgn}\left(\sin\left(\frac{\pi s_j(t)}{\alpha_j}\right)\right) - \rho.\end{aligned} \tag{15}$$

Note that, by the second condition in the theorem we have $\alpha_1 = \alpha, \alpha_2 = 2\alpha_1, \ldots, \alpha_n = 2\alpha_{n-1}$. Therefore, by the periodicity of the sinusoidal functions, it is suffices to consider the case when $s_i(t) \in (-\alpha_n, \alpha_n)$.

Suppose initially, $s_i(0) \in (0, \alpha)$, that is $s_i(0) > 0$. Then, there are two possible cases:

a) If $\dot{s}_i(t) < 0$, then $s(t)$ is a decreasing. Moreover, the sign of $\dot{s}(t)$ either remains same or changes after crossing $s_i(t) = k\alpha$, $k \in \mathbb{Z}$ and $k\alpha \in (-\alpha_n, \alpha_n)$. In the latter case, a sliding mode will exists at $s_i(t) = k\alpha$. Otherwise, the sign will change at $s_i(t) = -\alpha_n$ and a sliding mode will exists at $s_i(t) = -\alpha_n$.

b) Similarly, if $\dot{s}_i(t) > 0$, then $s(t)$ is increasing function. Moreover, the sign of $\dot{s}(t)$ either remains same or changes after crossing a point where $s_i(t) = k\alpha$, $k \in \mathbb{Z}$. In the latter case, a sliding mode will exists at $s_i(t) = k\alpha$. Otherwise, the sign will change at $s_i(t) = \alpha_n$ and a sliding mode will exists at $s_i(t) = \alpha_n$. ■

Note that, unlike the classical sliding mode, the ES controllers are sliding on a number of surfaces during the optimization process.

In sliding mode, the switching surface $s(t)$ will be constant and consequently the cost function $J(\theta(t))$ will follow the monotonic increasing function towards the maximum. Note that, after entering the region (13), it is possible that either the system output stays inside that region or goes through it. In the latter case, another sliding mode will happen and the system will enter the region again on the sliding mode.

Example 1: To illustrate the idea of the existence of sliding mode, we study the case of single variable, i.e. $n = 1$, as an example.

Consider a single variable cost function. The switching function is given by

$$s(t) = \hat{J}(\theta(t)) - g(t).$$

The time derivative of $s(t)$ is:

$$\begin{aligned}\dot{s}(t) &= \frac{\partial \hat{J}(\theta(t))}{\partial \theta}\dot{\theta}(t) - \rho. \\ &= -\frac{\partial \hat{J}(\theta(t))}{\partial \theta} \ k \ \mathrm{sgn}\left(\sin\left(\frac{\pi s(t)}{\alpha}\right)\right) - \rho.\end{aligned} \tag{16}$$

By the periodicity of the sinusoidal, it is suffices to consider the case when $s(t) \in (-\alpha, \alpha)$. Suppose initially we have $0 < s(0) < \alpha$ and that conditions in theorem 1 are satisfied.

a) If $\frac{\partial \hat{J}(\theta(t))}{\partial \theta} > 0$, then

$$\dot{s}(t) = -\frac{\partial \hat{J}(\theta(t))}{\partial \theta} \ k \ \mathrm{sgn}\left(\sin\left(\frac{\pi s(t)}{\alpha}\right)\right) - \rho < 0.$$

and therefore, $s(t) \to 0$.

When $-\alpha < s(t) < 0$ we have

$$\dot{s}(t) = -\frac{\partial \hat{J}(\theta(t))}{\partial \theta} k\mathrm{sgn}\left(\sin\left(\frac{\pi s(t)}{\alpha}\right)\right) - \rho > 0.$$

and therefore, $s(t) \to 0$. Hence, the system will slide on $s(t) = 0$.

b) On the other hand, If initially, $\frac{\partial \hat{J}(\theta(t))}{\partial \theta} < 0$. We have

$$\dot{s}(t) = -\frac{\partial \hat{J}(\theta(t))}{\partial \theta} k \ \mathrm{sgn}\left(\sin\left(\frac{\pi s(t)}{\alpha}\right)\right) - \rho > 0$$

and $s(t) \to \alpha$. When $\alpha < s(t) < 2\alpha$, we have

$$\dot{s}(t) = -\frac{\partial \hat{J}(\theta(t))}{\partial \theta} \ k \ \mathrm{sgn}\left(\sin\left(\frac{\pi s(t)}{\alpha}\right)\right) - \rho < 0.$$

So $s(t) \to \alpha$. Hence, the system will slide on $s(t) = \alpha$. Note that, by the symmetry of the sinusoidal function, the same results hold if $-\alpha < s(0) < 0$. Moreover, by the periodicity of the sinusoidal, it can be easily seen that for any $s(0) \in \mathbb{R}$, the system will slide on the nearest manifold $s(t) = n\alpha$ with:

- $n = 2K$ when $\frac{\partial \hat{J}(\theta(t))}{\partial \theta_i} > 0$.
- $n = 2K + 1$ when $\frac{\partial \hat{J}(\theta(t))}{\partial \theta_i} < 0$.

Where $n, K \in \mathbb{Z}$. Thus, sliding mode existence has been shown for the case when $n = 1$ which illustrate the idea of existence for the general case with n dimensional function. To show that in sliding mode $\hat{J}(\theta(t))$ is increasing, we will prove that $\theta(t)$ converges to the optimal point $\theta^\star$ asymptotically. According to theorem 1, sliding mode will be reached in finite time. Without loss of generality, suppose that at the time $t = t_{ri}$ the sliding surface s(t)=0 is reached. In sliding mode, the equivalent control v_{eq} can be obtained by solving the equation $\dot{s}(t) = 0$ for $v(t)$. That is,

$$\dot{s}(t) = \frac{\partial \hat{J}}{\partial \theta(t)} v_{eq}(t) - \rho = 0, \tag{17}$$

which implies,

$$v_{eq}(t) = \frac{\rho}{\frac{\partial \hat{J}}{\partial \theta(t)}}. \tag{18}$$

Furthermore, note that under the assumption 1 we have

$$(\theta(t) - \theta^\star)\frac{\partial \hat{J}}{\partial \theta(t)} < 0. \tag{19}$$

Let $\hat{\theta}(t) = \theta(t) - \theta^\star$, the time derivative of the function $\hat{\theta}(t)$ is

$$\dot{\hat{\theta}}(t) = v_{eq}(t). \tag{20}$$

From (18) and (19), it follows that

$$\hat{\theta}(t)\dot{\hat{\theta}}(t) < 0. \tag{21}$$

That is, $\theta(t) \to \theta^\star$ and $J(\theta(t))$ is increasing towards a neighborhood of the maximum, which is characterized by the region (13) as long as sliding mode exists.

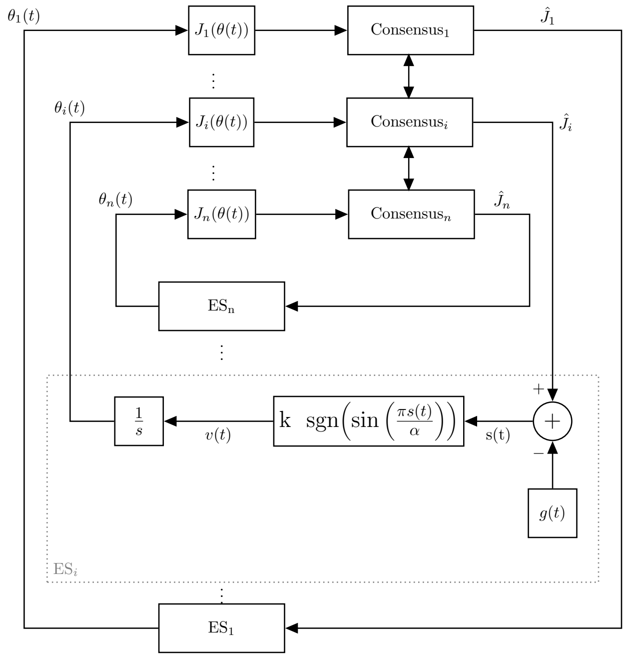

Fig. 3. Distributed Cooperative Extremum Seeking Control implementation scheme.

V. Application Example: Wind Farm Optimization

In this section, we illustrate the applicability of developed scheme to the problem of power maximization in a network of n wind turbines using the developed sliding mode based cooperative extremum seeking control. In wind turbines farm, wind turbines extract the kinetic energy from the wind flow, consequently, a wind turbine reduces the wind speed in the wake downstream of the wind turbine router. By considering the aerodynamic interaction among wind turbines, up to twenty-five percent of energy gain can be achieved under certain conditions.

We consider a wind farm consists of $n = 3$ wind turbines. Let θ_i be the control parameter of turbine i. Here, θ_i is the Axial Induction Factor (AIF) of turbine i and it takes values in $\left[0, \frac{1}{2}\right]$. For each wind turbine i, the power produce is given by

$$J_i(\theta) = \frac{1}{2}\rho_{air} A_i C_p(\theta_i) V_i(\theta)^3, \tag{22}$$

where ρ_{air} is the density of the air, A_i is the area swept by blades of turbine i, $C_p(\theta_i)$ is the power efficiency coefficient, and $V_i(\theta)$ is the wind speed at turbine i. Note that, in a wind farm, it is difficult to accurately model the coupling between the wind turbines, due to the interaction between the wind-turbine wake aerodynamics and the dynamics of the turbines. Therefore, it is assumed that agent i can only measure the power production of turbine i, and the problem is to maximize the total power production which is given by

$$P_{Total}(\theta_1, \theta_2, \ldots, \theta_n) = \sum_{i=1}^{n} J_i(\theta_1, \theta_2, \ldots, \theta_n) \tag{23}$$

The detailed model is described in [6]. Simulations are carried out using MATLAB. Note that, as shown in [6], the sup-optimal solution is achieved by setting all $\theta_i = \frac{1}{3}$. However, if we consider the optimal solution for the total power generated by all three turbines, we have:

$$\begin{aligned} P_{Total} &= J_1 + J_2 + J_3 \\ &= \frac{1}{2}\rho A (C_p(\theta_1) V_\infty^3 + C_p(\theta_2) V_2(\theta_1)^3 \\ &\quad + C_p(\theta_3) V_3(\theta_1, \theta_2)^3). \end{aligned} \tag{24}$$

Optimizing the above equation using numerical optimizer yields $\theta^* = (0.232, 0.208, 0.333)$. Fig. 4 shows the convergence of the consensus dynamic outputs. After reaching an agreement between the agents, each agent starts to optimize the global function using its own decision variable. The consensus parameters where chosen much larger than the controller gains k_i to ensure that the steady state is reached in a short time. The ES outputs reach the vicinity of the optimum after sliding on different surfaces as shown in Fig. 7. Unlike the classical sliding mode, the ES controllers are sliding on a number of surfaces during the optimization process. Fig. 5 demonstrates that starting, initially, with optimal setting for individual turbine (greedy policy) $\theta(0) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, the proposed control steers the farm to a vicinity of the maximum power generation. Since this is a derivative-free optimization, the ES controllers can only identify a vicinity of the optimal solution. The observed chattering can be reduced by lowering both parameters α_i and k_i, however, it will result in a slow convergence [4]. Furthermore, the convergence of the axial induction factor is demonstrated in Fig. 6. Different frequency is utilized, by selecting different α_i, in accordance to theorem 1. It is worth mentioning that there is also a trade-off between the convergence speed and the speed of the consensus dynamics.

VI. Conclusion

In this paper, a sliding mode based ES control has been proposed for solving a class of distributed optimization in a cooperative fashion. The strategy of introducing a consensus dynamic to communicate information about the global cost function has been considered to solve this class of problems. Stability and convergence analysis are discussed and sufficient conditions under which the overall scheme converges to the optimal solution were driven. The scheme, successfully, implemented to solve the problem of wind farm power maximization. The results obtained show the effectiveness of this technique in solving such problem and fast convergence of the algorithm to the unknown optimum was observed. Future research will include the development of ES controller in the presence of time delay as well as the development of distributed ES for constrained optimization.

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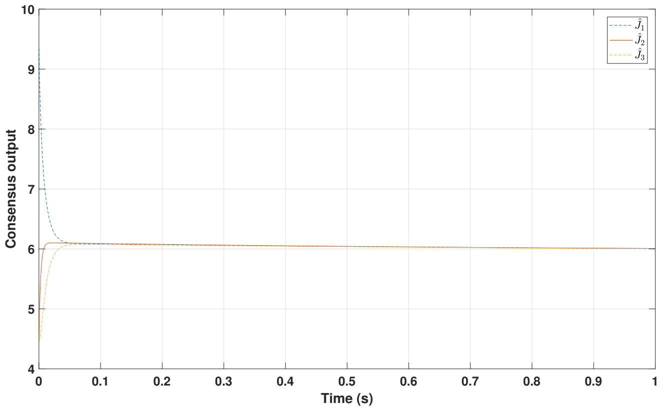

Fig. 4. Convergence of consensus dynamic outputs.

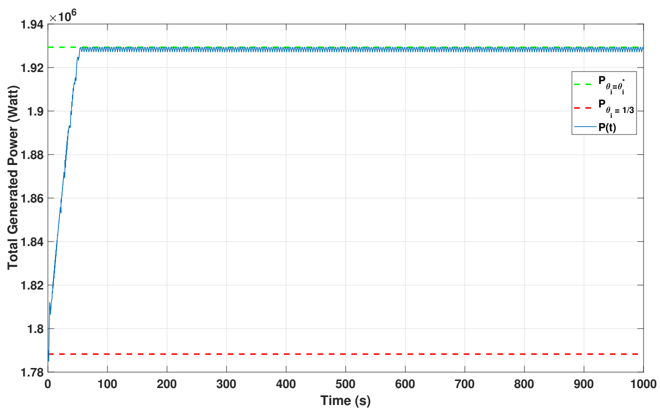

Fig. 5. Total power generated by the Farm.

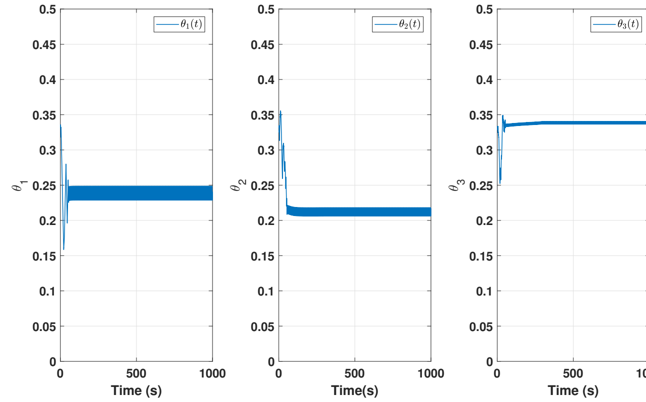

Fig. 6. Convergence of the axial induction factor for each turbine.

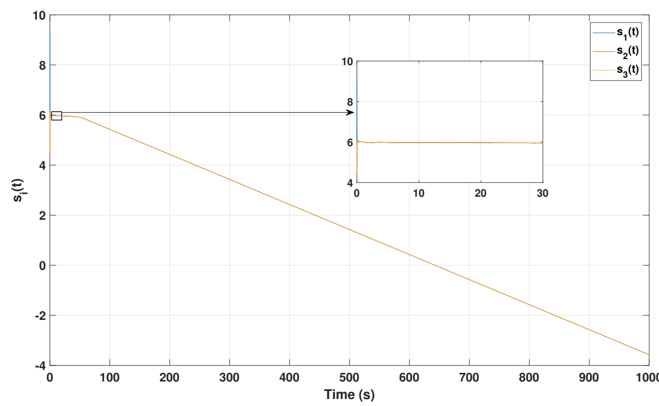

Fig. 7. Switching functions $s_i(t)$ for each agent

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