Math 2153

September 22

§ 13.1–13.3

1: Plane in 3D space

One point on the plane (x_0, y_0, z_0)

normal vector $\vec{n} = \langle a, b, c \rangle$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Line in 3D space

one point one the line (x_0, y_0, z_0)

director $\vec{v} = \langle a, b, c \rangle$

$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t \cdot \langle a, b, c \rangle$

2 Surfaces in 3D : $z = f(x,y)$ or $F(x,y,z)=0$

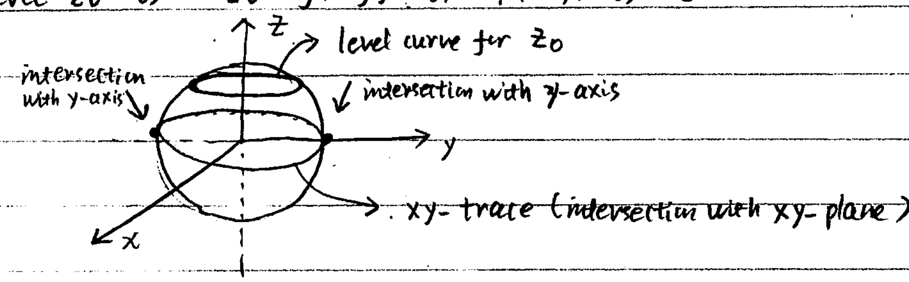
Def: Intersertions with x-axis, let $y=z=0$ and solve for x ; Similar Def for y-, z-axis

Def: Xy-trace: intersections with xy-plane. Let $z=0$ & find the relation of x and y.

Similar Def for yz-trace, xz-trace.

Def: Level curve for level z_0 is $z_0 = f(x,y)$ or $F(x,y,z_0)=0$

Demenstraction :



3 Functions with two or more variables

① Domain: the set of variables that make the function meaningful.

Common questions involve: a. Denominator $\neq 0$ e.g. $f(x,y) = \frac{1}{x-y}$: $\{(x,y) \mid x \neq y\}$

b. Squere root. e.g: $\sqrt{x-y}$: $\{(x,y) \mid x-y \geq 0\}$

c. Log function: e.g: $\ln(x-y)$: $\{(x,y) \mid x-y > 0\}$

☆ ② Limit $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$. Standard Procedure for this course.

Step 1: Check if (x_0, y_0) is on the boundary of domain. If not, plug in (x_0, y_0) and evaluate f at the point

Step 2: If (x_0, y_0) is on the boundary (i.e. you can't plug in), then simplify $f(x,y)$ and try again. (Techniques: factorization, rationalization)

Step 3: If step 2 fails, try 2-path test to show the limit Does Not Exist.

Two path test: find two paths through (x_0, y_0) and evaluate $\lim f(x,y)$ along different paths to get two different limits.

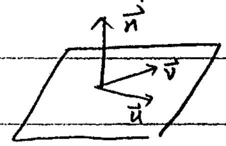
(Common questions: if $(x_0, y_0) = (0,0)$, then try $x=0$, $y=0$, $y=x$, $y=k \cdot x$, $y=kx^2$

③ Continuity.

Examples:

1. Find a plane that is parallel to $\vec{u}=\langle 2,0,2\rangle$ and $\vec{v}=\langle 0,1,2\rangle$, passing through $(4,0,1)$

Answer:



Normal vector: $\vec{n}=\vec{u}\times\vec{v}$!

$$\vec{u}\times\vec{v}=\begin{vmatrix} i & j & k \\ 2 & 0 & 2 \\ 0 & 1 & 2 \end{vmatrix} = -2\vec{i}-4\vec{j}+2\vec{k} = \langle -2,-4,2\rangle$$

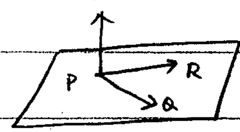
Plane: $-2(x-4)-4(y-0)+2(z-1)=0$

or $(x-4)+2y-(z-1)=0$

or $x+2y-z-3=0$ #

• More Practice: Find a plane through $\overset{P}{(4,2,3)}$ $\overset{Q}{(0,0,-2)}$ $\overset{R}{(-2,3,2)}$

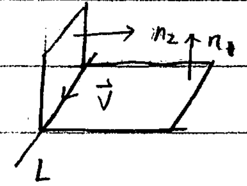
Hint:



generate $\vec{u}=\vec{PQ}$ $\vec{n}=\vec{u}\times\vec{v}$

$\vec{v}=\vec{PR}$

• More Practice: intersection line of two planes.



the direction of L is $\vec{v}=\vec{n}_1\times\vec{n}_2$

One point on L: find a special point on both planes.

The instructor solved a similar problem in class

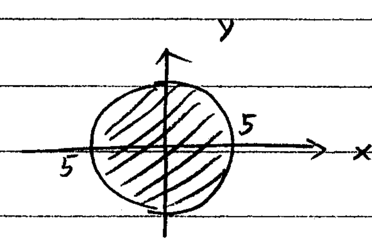
Also, check example 6 in §13.1

2. $f(x,y)=\sqrt{50-2x^2-2y^2}$

① Find the domain and sketch it

Answer: $50-2x^2-2y^2\geq 0 \iff x^2+y^2\leq 25$

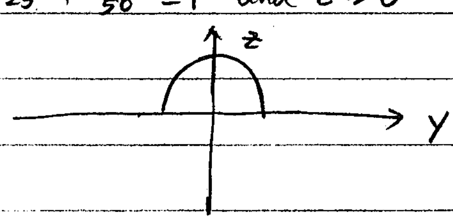
Domain: $\{(x,y) \mid x^2+y^2\leq 25\}$



② Find the yz-trace and sketch it.

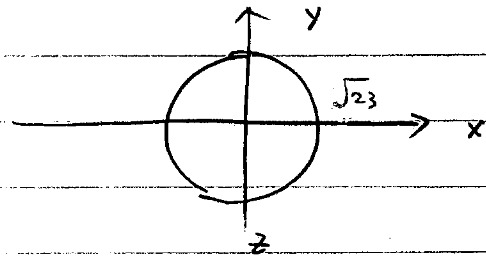
Answer: Let $x=0$. $z=\sqrt{50-2y^2} \iff z^2+2y^2=50$ and $z\geq 0$

$\iff \frac{y^2}{25}+\frac{z^2}{50}=1$ and $z\geq 0$

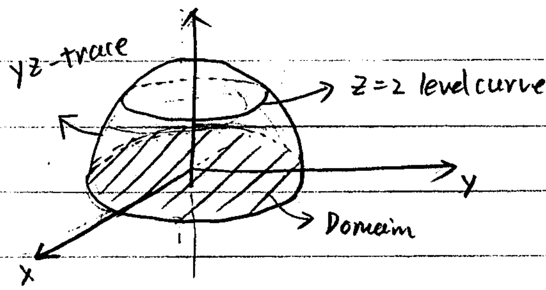


③ Find the level curve for $z=2$

Answer: $z=\sqrt{50-2x^2-2y^2} \Leftrightarrow 4=50-2x^2-2y^2 \Leftrightarrow x^2+y^2=23=(\sqrt{23})^2$



Note:



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3. Limits

① $\lim_{(x,y)\to(1,5)} \frac{-3x^2-4y^2-4}{x^2+y^2-2}$

Answer: step 1. ✓ $\lim_{(x,y)\to(1,5)} \frac{-3x^2-4y^2-4}{x^2+y^2-2} = \frac{-3\cdot1^2-4\cdot5^2-4}{1^2+5^2-2} = \frac{-107}{24}$

② $\lim_{(x,y)\to(3,13)} \frac{\sqrt{x+y}-4}{x+y-16}$

Answer: step 1 × cannot plug in

step 2: Simplify. $\lim_{(x,y)\to(3,13)} \frac{(\sqrt{x+y}-4)(\sqrt{x+y}+4)}{x+y-16\ (\sqrt{x+y}+4)} = \lim_{(x,y)\to(3,13)} \frac{\cancel{x+y-16}}{\cancel{(x+y-16)}(\sqrt{x+y}+4)}$

$= \lim_{(x,y)\to(3,13)} \frac{1}{\sqrt{x+y}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{8}$

③ $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$

Answer: step 1. × cannot plug in

step 2. $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(0,0)} \frac{\cancel{(x-y)}(x+y)}{\cancel{x-y}} = \lim_{(x,y)\to(0,0)} x+y = 0$

④ $\lim_{(x,y)\to(0,0)} \frac{4xy}{3x^2+y^2}$

Answer: step 1 × cannot plug in step 2 cannot simplify

step 3: Pick $x=0$ path. $\lim_{y\to0} \frac{0}{0+y^2} = 0$

Pick $x=y$ path. $\lim_{x\to0} \frac{4x^2}{3x^2+x^2} = \lim_{x\to0} \frac{4}{4} = 1$

> Different.

By two path test, the limit DNE.

(5). $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$

Answer: step 1 × cannot plug in step 2 cannot simplify

step 3 Pick $x=0$ $\lim_{y\to 0} \frac{0}{0+y^2} = 0$

Pick $y = x^3$ $\lim_{x\to 0} \frac{x^6}{x^6+x^6} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$ > different.

By two path test, the limit DNE. #