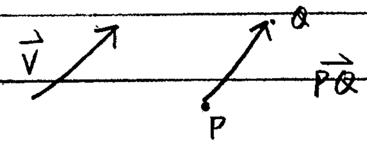
Aug. 25. 2016

§ 12.1 Vectors in the Plane

1. Vector: ① direction ② length (magnitude): $|\vec{v}|$

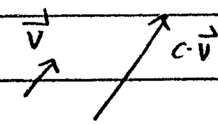


Unit vector: length $=1$

e.g.: unit vector in the direction of $\vec{v}$: $\dfrac{\vec{v}}{|\vec{v}|}$

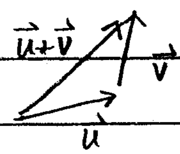
2. Operations:

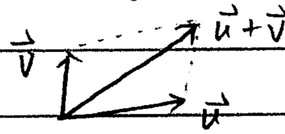
① Scalar Multiples: $c \cdot \vec{v}$, c is a constant.



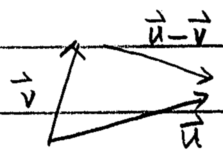
② Addition and substraction:

Addition: Triangle Rule. Parallelogram Rule





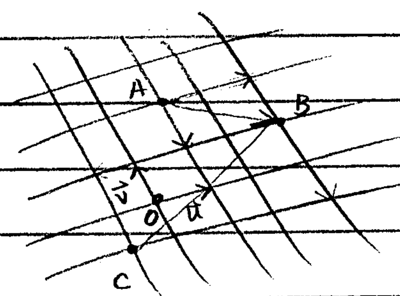
Subtraction:

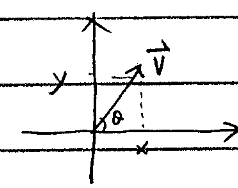


e.g.: write the following vector as scalar multiples of $\vec{u}$ and $\vec{v}$ (HW #2, #3)

$\overrightarrow{AB}$

$\overrightarrow{BC}$

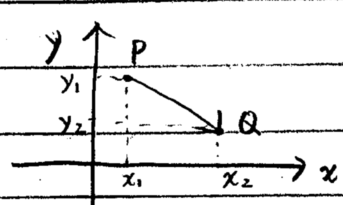




Answer: $\overrightarrow{AB} = 2\vec{u} - \vec{v}$

$\overrightarrow{BC} = -4\vec{u} - 2\vec{v}$

3. Coordinates: ① $\vec{v} = \langle x, y \rangle$ $|\vec{v}| = \sqrt{x^2+y^2}$ and $\vec{v} = \langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$



or: $P = (x_1, y_1)$ $Q(x_2, y_2)$

$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$ angle bracket

② $|\overrightarrow{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

③ Operations: $\vec{u} = \langle x_1, y_1 \rangle$ $\vec{v} = \langle x_2, y_2 \rangle$

$c \cdot \vec{u} = \langle c \cdot x_1, c \cdot y_1 \rangle$ $\vec{u} + \vec{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$ $\vec{u} - \vec{v} = \langle x_1 - x_2, y_1 - y_2 \rangle$

e.g. $P(3,2)$ $Q(4,2)$ $R(-6,-1)$ Find $|\overrightarrow{PQ}|$ $|\overrightarrow{RQ}|$

Answer: $\overrightarrow{PQ} = \langle 4-3, 2-2 \rangle = \langle 1, 0 \rangle$ $\quad |\overrightarrow{PQ}| = \sqrt{1+0} = 1$

$\overrightarrow{RQ} = \langle 4-(-6), 2-(-1) \rangle = \langle 10, 3 \rangle$ $\quad |\overrightarrow{RQ}| = \sqrt{10^2+3^2} = \sqrt{109}$

e.g. $\vec{u} = \langle 3, -4 \rangle$ $\vec{v} = \langle 1, 1 \rangle$ $\vec{w} = \langle -1, 0 \rangle$ Find $|3\vec{u}+\vec{v}-2\vec{w}|$

Answer: $3\vec{u}+\vec{v}-2\vec{w} = \langle 3\cdot 3+1-2(-1),\ 3\times(-4)+1-0\times 2 \rangle$

$= \langle 12, -11 \rangle$

$|3\vec{u}+\vec{v}-2\vec{w}| = \sqrt{12^2+(-11)^2} = \sqrt{144+121} = \sqrt{265}$ #

4 Applications

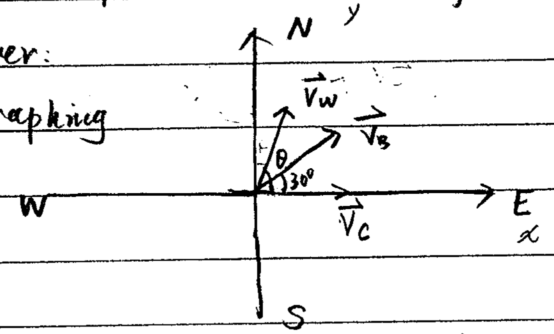
e.g. Boat in wind.

A boat floats in a current that flows due east at 1 m/s. Due to wind, the actual speed relative to shore is $\sqrt{3}$ m/s, in a direction 30° north of east.

Find speed and direction of wind.

Answer:

Graphing



Current $|\vec{V_c}| = 1$ m/s

Boat $|\vec{V_B}| = \sqrt{3}$ m/s

Wind $|\vec{V_w}| = ?$

Recall $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$.

Relation: $\vec{V_B} = \vec{V_C} + \vec{V_W}$

$\vec{V_C} = \langle 1, 0 \rangle$ $\quad \vec{V_B} = \langle \sqrt{3}\cos\frac{\pi}{6}, \sqrt{3}\sin\frac{\pi}{6} \rangle = \langle \frac{3}{2}, \frac{\sqrt{3}}{2} \rangle$

$\vec{V_W} = \vec{V_B} - \vec{V_C} = \langle \frac{3}{2}, \frac{\sqrt{3}}{2} \rangle - \langle 1, 0 \rangle = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

$\tan\theta = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ $\quad \theta = \frac{\pi}{3} = 60^\circ$ north of east.

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