Math 2153 - Autumn 2016 Name (Please print): ______________

Quiz 8 Recitation time (Please circle): 1:50 3:00 4:10

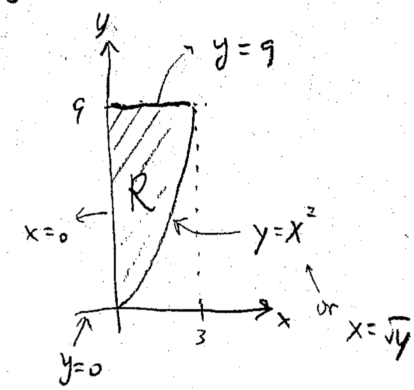
SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [6 pts] Evaluate the following double integral **by first reversing the order of integration**

$$\int_0^3 \int_{x^2}^9 \sqrt{y}\cos(y)\mathrm{d}y\mathrm{d}x.$$

Step 1. Figure out the region of integration.

$$R = \{ (x,y) \mid 0 \le x \le 3 , \quad x^2 \le y \le 9 \}$$



Step 2. Reverse the order of integration.

$$I = \int_0^3 \int_{x^2}^9 \sqrt{y}\cos(y)\,dy\,dx$$

$$= \int_0^9 \int_0^{\sqrt{y}} \sqrt{y}\cos(y)\,dx\,dy$$

Step 3. Evaluate the integral

$$I = \int_0^9 \int_0^{\sqrt{y}} \sqrt{y}\cos(y)\,dx\,dy$$

$$= \int_0^9 \left(\left(\sqrt{y}\cos(y)\cdot x\right)\Big|_{x=0}^{x=\sqrt{y}} \right) dy = \int_0^9 y\cos(y)\,dy \quad \left(\text{Use integration by parts}\right)$$

$$\begin{pmatrix} u = y, \; dv = \cos(y)dy \\ du = dy, \; v = \sin(y) \end{pmatrix} \quad = y\sin(y)\Big|_0^9 - \int_0^9 \sin y\,dy$$

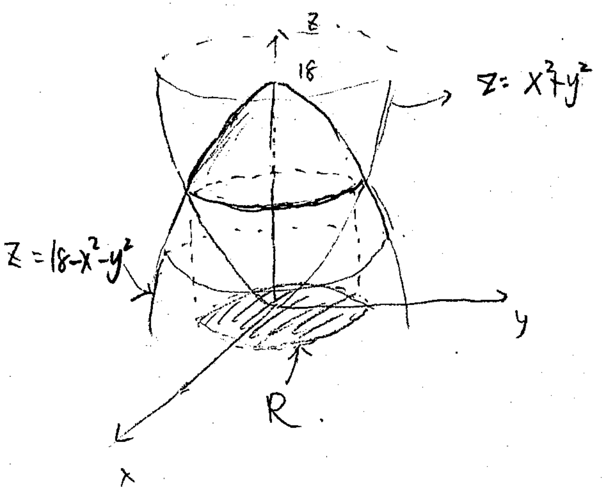
$$\int u\,dv = uv - \int v\,du \quad = 9\sin(9) - 0\cdot\sin(0) + \cos(y)\Big|_0^9$$

$$= 9\sin(9) + \cos(9) - 1$$

(There is another problem on the back.)

but do not evaluate

Problem 2 [4 pts] Set up a double integral (in Cartesian coordinates) which represents the volume of the solid region bounded by the paraboloids $z = x^2+y^2$ and $z = 18-x^2-y^2$.



The intersection of the paraboloids :

$$z = x^2+y^2 = 18-x^2-y^2$$

$$2x^2+2y^2 = 18$$

$$\begin{cases} x^2+y^2 = 9 = 3^2 \\ z = 9 \end{cases}$$

So the region $R = \{(x,y) \mid x^2+y^2 \le 3^2\}$

(or) $= \{(x,y) \mid -3 \le x \le 3,\ -\sqrt{9-x^2} \le y \le \sqrt{9-x^2}\}$

(or) $= \{(x,y) \mid -3 \le y \le 3,\ -\sqrt{9-y^2} \le x \le \sqrt{9-y^2}\}$

and

Volume of the solid region

$$= \iint_R (18-x^2-y^2) - (x^2+y^2)\, dA$$

or) $$= \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (18-x^2-y^2)-(x^2+y^2)\, dy\, dx$$

or) $$= \int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} (18-x^2-y^2)-(x^2+y^2)\, dx\, dy$$