Math 2153 - Autumn 2016 Name (Please print): ____________________

Quiz 7 Recitation time (Please circle): 1:50 3:00 4:10

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

**Problem 1** [10 pts] Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = 10x - 6y + 8z$ subject to the constraint $x^2 + y^2 + z^2 = 50$.

Let $g(x,y,z) = x^2+y^2+z^2-50$.

Set up the Lagrange system

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$

$\nabla f = \langle 10, -6, 8 \rangle$, $\nabla g = \langle 2x, 2y, 2z \rangle$

So the system becomes

$$\begin{cases} 10 = 2\lambda x & \cdots ① \\ -6 = 2\lambda y & \cdots ② \\ 8 = 2\lambda z & \cdots ③ \\ x^2+y^2+z^2-50=0 & \cdots ④ \end{cases}$$

The left sides of ①, ②, ③ imply that $\lambda \neq 0, x \neq 0, y \neq 0, z \neq 0$.

Approach 1 Eliminate $\lambda$:

$\frac{②}{①}$ gives $\frac{y}{x} = \frac{-6}{10}$, i.e., $y = -\frac{3}{5}x$ $\cdots$ ⑤

$\frac{③}{①}$ gives $\frac{z}{x} = \frac{8}{10}$, i.e., $z = \frac{4}{5}x$ $\cdots$ ⑥

(see the next page)

Pluggin ⑤ and ⑥ in ④ gives

$$x^2 + \frac{9}{25}x^2 + \frac{16}{25}x^2 - 50 = 0,$$

which gives $x^2 = 25$, i.e., $x = \pm 5$. Hence we have two points (by ⑤ and ⑥)

$$(5, -3, 4) \quad \text{and} \quad (-5, 3, -4)$$

$f(5,-3,4) = 100$, $f(-5,3,-4) = -100$.

So $f$ attains a maximum 100 at $(5,-3,4)$
and a minimum $-100$ at $(-5,3,-4)$.

Approach 2. Solve for $\lambda$:

①, ②, ③ give

$$x = \frac{5}{\lambda}, \quad y = \frac{-3}{\lambda}, \quad z = \frac{4}{\lambda} \quad \cdots (*)$$

Plugging these in ④ gives

$$\frac{25}{\lambda^2} + \frac{9}{\lambda^2} + \frac{16}{\lambda^2} - 50 = 0$$

i.e.,

$$\frac{50}{\lambda^2} = 50, \quad \text{or} \quad \lambda^2 = 1.$$

So $\lambda = \pm 1$.

when $\lambda = 1$, $(*)$ gives a point $(5,-3,4)$

when $\lambda = -1$, $(*)$ gives a point $(-5,3,-4)$

$f(5,-3,4) = 100$ and $f(-5,3,-4) = -100$

So $f$ attains a maximum 100 at $(5,-3,4)$
and a minimum $-100$ at $(-5,3,-4)$