Math 2153 - AU 2016

Quiz 6

Due Monday Oct. 17 in lecture

Name and name.#(Please print): ____________________

Recitation teacher and time: ____________________

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [4 pts] Consider the function $f(x,y) = 5\sqrt{x^2+y^2}$.

a. [1 pt] Find the gradient of the function $f(x,y)$.

The gradient is $\nabla f(x,y) = \langle f_x, f_y \rangle$

$$f_x = 5 \cdot \frac{2x}{2\sqrt{x^2+y^2}} = \frac{5x}{\sqrt{x^2+y^2}}, \quad f_y = 5 \cdot \frac{2y}{2\sqrt{x^2+y^2}} = \frac{5y}{\sqrt{x^2+y^2}}$$

So

$$\nabla f(x,y) = \left\langle \frac{5x}{\sqrt{x^2+y^2}}, \frac{5y}{\sqrt{x^2+y^2}} \right\rangle$$

b. [1 pt] Find the equation of the tangent plane of the surface $z = f(x,y)$ at $(3,-4)$.

The tangent plane at $(3,-4)$ has a normal vector

$$\vec{n} = \langle \nabla f(3,-4), -1 \rangle$$

$$= \langle 3, -4, -1 \rangle$$

So the equation of the tangent plane is

$$3(x-3) + (-4)(y-(-4)) + (-1)(z-25) = 0$$

or $3x - 4y - z = 0$

or $z = 3x - 4y$

c. [2 pts] Find the linear approximation $L(x,y)$ of f at $(3,-4)$ and estimate $f(3.06,-3.92)$ by using $L(x,y)$.

The linear approximation $L(x,y)$ of f at $(3,-4)$ is the tangent plane $z = 3x-4y$, i.e., $L(x,y) = 3x - 4y$.

So $f(3.06,-3.92) \approx L(3.06,-3.92) = 3 \times 3.06 + 4 \times 3.92 = 24.86$

(Note: By using your calculator, the function value of $f(3.06,-3.92)$ is $24.8646\cdots\cdots$)

(There are problems on the next page!)

Problem 2 [6 pts] Consider the function $f(x, y) = x^2 + xy^2 - 2x + 1$.

a. [2 pts] Find all critical point(s) of f.

Set $\begin{cases} f_x = 2x + y^2 - 2 = 0 \quad \cdots (1) \\ f_y = 2xy = 0 \quad \cdots (2) \end{cases}$ to solve for x, y.

By (2), $x = 0$ or $y = 0$.

If $x = 0$ then (1) $\Rightarrow y^2 = 2 \iff y = \pm\sqrt{2}$

If $y = 0$ then (1) $\Rightarrow x = 1$.

So the critical points of f are $(0, \sqrt{2})$, $(0, -\sqrt{2})$, and $(1, 0)$

b. [4 pts] Use The Second Derivative Test to determine whether EACH critical point you find in part (a) corresponds to a local max, a local min, or a saddle point.

$f_{xx} = 2$, $f_{yy} = 2x$, $f_{xy} = 2y$.

So the discriminant function

$$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2 = 4x - 4y^2$$

$D(0, \sqrt{2}) = D(0, -\sqrt{2}) = 4 \cdot 0 - 4 \cdot 2 = -8 < 0$

So the 2nd derivative test $\Rightarrow (0, \pm\sqrt{2})$ correspond to saddle points of f

$D(1, 0) = 4 \cdot 1 - 4 \cdot 0 = 4 > 0$

$f_{xx}(1, 0) = 2 > 0$

So the 2nd derivative test $\Rightarrow (1, 0)$ corresponds to a local min of f.