

Quiz 5

Recitation time (Please circle): 1:50 3:00 4:10

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [4 pts] Find an equation of the plane that is parallel to the vectors $\langle 0, 1, 1 \rangle$ and $\langle 2, 0, 1 \rangle$, passing through the point $(1, 0, 1)$.

A normal vector of the plane is

$$\vec{n} = \langle 0, 1, 1 \rangle \times \langle 2, 0, 1 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} \vec{k}$$

$$= \vec{i} + 2\vec{j} - 2\vec{k} = \langle 1, 2, -2 \rangle$$

We know a point $(1, 0, 1)$ on the plane.

So an equation of the plane is

$$1 \cdot (x-1) + 2(y-0) + (-2)(z-1) = 0$$

or

$$x + 2y - 2z = -1$$

(There are problems on the back!)

Problem 2

a. [3 pts] Find the limit $\lim_{(x,y) \rightarrow (1,-1)} \frac{x^2 + xy}{x^2 - y^2}$ or show it does not exist.

Direct plug-in of $(1, -1)$ gives " $\frac{0}{0}$ ". So try simplification.

$$\lim_{(x,y) \rightarrow (1,-1)} \frac{x^2 + xy}{x^2 - y^2} = \lim_{(x,y) \rightarrow (1,-1)} \frac{x(x+y)}{(x-y)(x+y)}$$

$$= \lim_{(x,y) \rightarrow (1,-1)} \frac{x}{x-y}$$

$$(\text{plug-in}) = \frac{1}{1 - (-1)} = \frac{1}{2}$$

b. [3 pts] Find the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$ or show it does not exist.

Direct plug-in of $(0,0)$ gives " $\frac{0}{0}$ ". Simplification of $\frac{x^2 y}{x^4 + y^2}$ does not work.

So try the Two-Path Test.

If you use linear paths through $(0,0)$, the resulting limit is always 0.

So can try quadratic paths $y = mx^2$ with varying m :

$$\lim_{\substack{(x,y) \rightarrow (0,0) \\ \text{along } y=mx^2}} \frac{x^2 y}{x^4 + y^2} = \lim_{x \rightarrow 0} \frac{x^2 (mx^2)}{x^4 + (mx^2)^2} = \lim_{x \rightarrow 0} \frac{m x^4}{x^4 + m^2 x^4} = \frac{m}{1+m^2}$$

setting $m=0$ and $m=1$ gives two different limits 0 and $\frac{1}{2}$, say.

So the Two-path Test \Rightarrow the limit in question does not exist.