

Quiz 4

Recitation time (Please circle): 15:00 16:10 17:20

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [10 pts]

Consider the parametric curve $\mathbf{r}(t) = \langle t, \sin(3t), \cos(3t) \rangle$ ($\pi \leq t \leq 2\pi$).

(a) [3 pts] Find the unit tangent vector $\mathbf{T}(t)$.

$$\vec{r}'(t) = \langle 1, 3\cos(3t), -3\sin(3t) \rangle$$

$$\begin{aligned} |\vec{r}'(t)| &= \sqrt{1^2 + 3^2\cos^2(3t) + (-3)^2\sin^2(3t)} = \sqrt{1 + 9(\cos^2(3t) + \sin^2(3t))} \\ &= \sqrt{10} \end{aligned}$$

So
$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{1}{\sqrt{10}} \langle 1, 3\cos(3t), -3\sin(3t) \rangle$$

(b) [3 pts] Find the curvature $\kappa(t)$.

$$\vec{T}'(t) = \frac{1}{\sqrt{10}} \langle 0, -9\sin(3t), -9\cos(3t) \rangle$$

$$\begin{aligned} |\vec{T}'(t)| &= \frac{1}{\sqrt{10}} \sqrt{0^2 + (-9)^2\sin^2(3t) + (-9)^2\cos^2(3t)} \\ &= \frac{1}{\sqrt{10}} \sqrt{81(\sin^2(3t) + \cos^2(3t))} \\ &= \frac{9}{\sqrt{10}} \end{aligned}$$

So

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{\frac{9}{\sqrt{10}}}{\sqrt{10}} = \frac{9}{\sqrt{10} \cdot \sqrt{10}} = \frac{9}{10}$$

(There is a problem on the back!)

Still use the curve $\mathbf{r}(t) = \langle t, \sin(3t), \cos(3t) \rangle$ ($\pi \leq t \leq 2\pi$).

(c) [4 pts] Determine whether $\mathbf{r}(t)$ uses the arc length as a parameter. If not, find a description that uses the arc length parameter s and specify the range of s .

By computation in (a),

$$|\vec{r}'(t)| = \sqrt{10} \neq 1$$

So $\vec{r}(t)$ does NOT use arclength as a parameter.

The arc length function

$$s(t) = \int_{\pi}^t |\vec{r}'(u)| du = \sqrt{10} t - \sqrt{10} \pi$$

$$\text{Set } s = \sqrt{10} t - \sqrt{10} \pi \quad \text{and get } t = \frac{s + \sqrt{10} \pi}{\sqrt{10}} = \frac{s}{\sqrt{10}} + \pi$$

Replace t by $\frac{s}{\sqrt{10}} + \pi$ in $\vec{r}(t)$ to get a new description

$$\vec{R}(s) = \left\langle \frac{s}{\sqrt{10}} + \pi, \sin\left(3\left(\frac{s}{\sqrt{10}} + \pi\right)\right), \cos\left(3\left(\frac{s}{\sqrt{10}} + \pi\right)\right) \right\rangle$$

The range of s is $0 \leq s \leq \sqrt{10} \pi$.