Math 2153 - Autumn 2016 Name: ____________

Quiz 3 Recitation time (Please circle): 1:50 3:00 4:10

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [4 pts]
Consider the parametrized curve $\mathbf{r}(t) = \langle e^{2t}, 2e^{2t}, 2e^{-3t} \rangle$, for $0 \le t \le 1$.

(a) [1 pt] Compute $\mathbf{r}'(t)$.

$$\vec{r}'(t) = \langle 2e^{2t}, 4e^{2t}, -6e^{-3t} \rangle$$

(b) [1 pt] Find a tangent vector at $t = \ln 2$.

$$\vec{r}'(\ln 2) = \langle 2e^{2\ln 2}, 4e^{2\ln 2}, -6e^{-3\ln 2} \rangle$$

$$= \langle 2e^{\ln 2^2}, 4e^{\ln 2^2}, -6e^{\ln 2^{-3}} \rangle$$

$$= \langle 2 \cdot 2^2, 4 \cdot 2^2, -6 \cdot 2^{-3} \rangle$$

$$= \langle 8, 16, -\tfrac{3}{4} \rangle$$

(c) [2 pts] Find the unit tangent vector $\mathbf{T}(t)$.

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\langle 2e^{2t}, 4e^{2t}, -6e^{-3t} \rangle}{\sqrt{4e^{4t} + 16e^{4t} + 36e^{-6t}}}$$

$$= \frac{\langle 2e^{2t}, 4e^{2t}, -6e^{-3t} \rangle}{\sqrt{20e^{4t} + 36e^{-6t}}}$$

Problem 2 [6 pts]
A spaceship is traveling with acceleration $\mathbf{a}(t) = \langle t, e^{-t}, 1\rangle$. At $t = 0$, the space ship was at initial point $\mathbf{r}(0) = \langle 4, 0, 0\rangle$ and had initial velocity $\mathbf{v}(0) = \langle 0, 0, 1\rangle$.

(a) [3 pts] Find the velocity of the spaceship, for $t \geq 0$.

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \left\langle \int t\,dt, \int e^{-t}\,dt, \int 1\,dt \right\rangle + \langle C_1, C_2, C_3\rangle$$

$$= \left\langle \tfrac{1}{2}t^2, -e^{-t}, t \right\rangle + \langle C_1, C_2, C_3\rangle$$

By $\vec{v}(0) = \langle 0, -1, 0\rangle + \langle C_1, C_2, C_3\rangle = \langle 0, 0, 1\rangle$,

$\langle C_1, C_2, C_3\rangle = \langle 0, 1, 1\rangle$.

So

$$\vec{v}(t) = \left\langle \tfrac{1}{2}t^2, -e^{-t}, t\right\rangle + \langle 0, 1, 1\rangle$$

$$= \left\langle \tfrac{1}{2}t^2, -e^{-t}+1, t+1\right\rangle$$

(b) [3 pts] Find the position of the spaceship, for $t \geq 0$.

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \left\langle \int \tfrac{1}{2}t^2\,dt, \int (-e^{-t}+1)\,dt, \int (t+1)\,dt \right\rangle$$

$$= \left\langle \tfrac{1}{6}t^3, e^{-t}+t, \tfrac{1}{2}t^2+t \right\rangle + \langle d_1, d_2, d_3\rangle,$$

By $\vec{r}(0) = \langle 0, 1, 0\rangle + \langle d_1, d_2, d_3\rangle = \langle 4, 0, 0\rangle$

$\langle d_1, d_2, d_3\rangle = \langle 4, -1, 0\rangle$

So

$$\vec{r}(t) = \left\langle \tfrac{1}{6}t^3+4, e^{-t}+t-1, \tfrac{1}{2}t^2+t \right\rangle$$