Math 2153 - Autumn 2016 Name (Please print): ______________________

Quiz 2 Recitation time (Please circle): 1:50 3:00 4:10

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [5 pts]

Consider the vectors $\mathbf{u} = \langle 5, 1, -5 \rangle$ and $\mathbf{v} = \langle -1, 1, -2 \rangle$.

(a) [1 pt] Compute $\mathbf{u} \cdot \mathbf{v}$.

$$\vec{u} \cdot \vec{v} = 5 \cdot (-1) + 1 \cdot 1 + (-5) \cdot (-2)$$
$$= -5 + 1 + 10$$
$$= 6$$

(b) [2 pts] Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

$$\text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \right) \frac{\vec{v}}{|\vec{v}|}$$
$$= \left(\frac{6}{\sqrt{6}} \right) \frac{\vec{v}}{\sqrt{6}}$$
$$= \vec{v} = \langle -1, 1, -2 \rangle$$

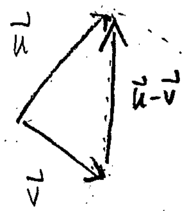
$$|\vec{v}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$$
$$= \sqrt{1+1+4} = \sqrt{6}$$

(c) [2 pts] Find $\text{scal}_{\mathbf{v}}\mathbf{u}$.

$$\text{scal}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

Problem 2 [5 pts]
Consider the vectors $\mathbf{u} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

(a) [3 pts] Find the area of the triangle whose sides are $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$.



A = Area of the triangle

$= \frac{1}{2}$ Area of the parallelogram spanned by $\vec{u}$ and $\vec{v}$

$= \frac{1}{2} |\vec{u} \times \vec{v}|$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -2 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -1 & -2 \\ 3 & -2 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & -2 \\ 1 & -2 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} \vec{k}$$

$$= ((-1)(-2) - (-2)\cdot 3)\,\vec{i} - (3\cdot(-2) - (-2)\cdot 1)\,\vec{j} + (3\cdot 3 - (-1\cdot 1))\,\vec{k}$$

$$= 8\vec{i} + 4\vec{j} + 10\vec{k}$$

So $A = \frac{1}{2} |\vec{u} \times \vec{v}| = \frac{1}{2}\sqrt{8^2 + 4^2 + 10^2} = \frac{1}{2}\sqrt{180} = 3\sqrt{5}$

(b) [2 pts] Find an equation of the line that passes through the point $(1, 2, -3)$ and is normal to both $\mathbf{u}$ and $\mathbf{v}$. (You may give either the vector-valued equation of the line or the parametric equation of the line.)

a direction vector of the line is $\vec{u} \times \vec{v} = \langle 8, 4, 10 \rangle$

$(1, 2, -3)$ is on the line.

So an equation for the line is

$$\vec{r}(t) = \langle 1, 2, -3 \rangle + t \langle 8, 4, 10 \rangle$$
$$= \langle 1 + 8t,\ 2 + 4t,\ -3 + 10t \rangle$$

or the parametric equations of the line are

$$\begin{cases} x = 1 + 8t \\ y = 2 + 4t \\ z = -3 + 10t \end{cases}$$