Math 2153 - Autumn 2016 Name (Please print): ____________

Quiz 11 Recitation time (Please circle): 1:50 3:00 4:10

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [4 pts] Convert the following line integral to an ordinary integral with respect to the parameter **and evaluate it**

$$\int_C (z - x)\mathrm{d}s,$$

where the curve C is the helix $\mathbf{r}(t) = \langle \underbrace{3\sin t}_{x}, \underbrace{3\cos t}_{y}, \underbrace{t}_{z} \rangle$ $(0 \le t \le 2\pi)$.

$$\vec{r}\,'(t) = \langle 3\cos t, -3\sin t, 1 \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{(3\cos t)^2 + (-3\sin t)^2 + 1^2} = \sqrt{10}$$

So

$$\int_C (z - x)\,ds = \int_0^{2\pi} (t - 3\sin t)\,|\vec{r}\,'(t)|\,dt$$

$$= \int_0^{2\pi} (t - 3\sin t)\sqrt{10}\,dt$$

$$= \sqrt{10}\left(\frac{t^2}{2} + 3\cos t\right)\Big|_0^{2\pi}$$

$$= \sqrt{10}\left(\frac{4\pi^2}{2} + 3\right) - \sqrt{10}\left(\frac{0^2}{2} + 3\right)$$

$$= \sqrt{10}\cdot 2\pi^2$$

(There is another problem on the back.)

Problem 2 Let $\varphi(x,y) = x^2 - y^2$.
(a) [2 pts] Find the gradient field $\mathbf{F}$ of $\varphi(x,y)$;
(b) [4 pts] Evaluate the line integral

$$\int_C \mathbf{F}\cdot\mathbf{T}ds,$$

where C is the semi-circle $\mathbf{r}(t) = \langle \underbrace{4\cos t}_{x}, \underbrace{4\sin t}_{y}\rangle$ $(0 \le t \le \pi)$.

a) $\vec{F} = \nabla\varphi = \langle \frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y} \rangle$

$= \langle 2x, -2y \rangle$

b) $\int_C \vec{F}\cdot\vec{T}\,ds = \int_C \vec{F}\cdot d\vec{r} = \int_0^\pi \vec{F}\cdot\vec{r}'(t)\,dt$

$\vec{r}'(t) = \langle -4\sin t, 4\cos t \rangle$

$\vec{F} = \langle 2\cdot 4\cos t, -2\cdot 4\sin t \rangle = \langle 8\cos t, -8\sin t \rangle$

$\vec{F}\cdot\vec{r}'(t) = 8\cos t\cdot(-4\sin t) + (-8\sin t)\cdot 4\cos t$

$= -32\sin t\cdot\cos t - 32\sin t\cdot\cos t = -64\sin t\cdot\cos t$

$= -32\sin(2t)$

So $\int_C \vec{F}\cdot\vec{T}\,ds = \int_0^\pi -32\sin(2t)\,dt$

$= 16\cos 2t \Big|_0^\pi$

$= 0$