Math 2153 - Autumn 2016 Name (Please print): _____

Quiz 1 Recitation time (Please circle): 1:50 3:00 4:10

SHOW ALL WORK FOR THE PROBLEMS!!! Unsupported answers might not receive full credit.

Problem 1 [6 pts]

Consider the vectors $\overrightarrow{OP} = \langle 3, 8, 12 \rangle$ and $\overrightarrow{OQ} = \langle 3, 9, 11 \rangle$.

(a) [2 pts] Find $\overrightarrow{PQ}$ and state your answer in the form of $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \langle 3-3, 9-8, 11-12 \rangle = \langle 0, 1, -1 \rangle$

$= \vec{j} - \vec{k}$

(b) [1 pt] Find the magnitude of $\overrightarrow{PQ}$.

$|\overrightarrow{PQ}| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}.$

(c) [3 pts] Find **two unit vectors** parallel to $\overrightarrow{PQ}$.

They are $\pm \dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \pm \dfrac{1}{\sqrt{2}} \langle 0, 1, -1 \rangle = \pm \langle 0, \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}} \rangle$

Problem 2 [4 pts]
Consider the vectors $\mathbf{u} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{j} - 3\mathbf{k}$

(a) [2 pts] Find the vector that is in the same direction as $\mathbf{u}$ but has the same magnitude as the vector $\mathbf{u} + \mathbf{v}$.

$\vec{u} + \vec{v} = 3\vec{i} - \vec{j} + \vec{k} + \vec{j} - 3\vec{k} = 3\vec{i} - 2\vec{k} = \langle 3, 0, -2 \rangle$

$|\vec{u} + \vec{v}| = \sqrt{3^2 + 0^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

Let $\vec{w}$ be the desired vector.

Then $|\vec{w}| = |\vec{u} + \vec{v}| = \sqrt{13}$ and $\vec{w}$ has the same direction as $\vec{u}$.

So $\vec{w} = \underbrace{|\vec{u} + \vec{v}|}_{\text{magnitude}} \underbrace{\frac{\vec{u}}{|\vec{u}|}}_{\text{direction}} = \frac{\sqrt{13}}{\sqrt{11}} \langle 3, -1, 1 \rangle$,

since $|\vec{u}| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11}$.

(b) [2 pts] Determine the value of x such that the points $(3, -1, 1)$, $(0, 1, -3)$, and $(-3, x, -7)$ are collinear (i.e. the points lie on a same line).

$A(3, -1, 1)$, $B(0, 1, -3)$, $C(-3, x, -7)$

$\overrightarrow{AB} = \langle 0-3, 1-(-1), -3-1 \rangle = \langle -3, 2, -4 \rangle$ ①

$\overrightarrow{BC} = \langle -3-0, x-1, -7-(-3) \rangle = \langle -3, x-1, -4 \rangle$ ②

A, B, C collinear $\Leftrightarrow$ $\overrightarrow{BC} = \lambda \overrightarrow{AB}$ for some number λ

① and ② $\Rightarrow \lambda = 1$, so $x - 1 = 2$ and $\underline{x = 3}$.