Math 1172 - Autumn 2015 Name: ____________________

Quiz 9 - In Class Recitation Instructor: ____________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [4 pts] Suppose $\vec{u} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{v} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$

$= \langle 1, -1, 1 \rangle$ $= \langle 2, 1, -3 \rangle$

a) [2 pts] Find $\vec{u} \times \vec{v}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \vec{k}$$

$$= (3-1)\vec{i} - (-3-2)\vec{j} + (1+2)\vec{k}$$

$$= 2\vec{i} + 5\vec{j} + 3\vec{k}$$

$$= \langle 2, 5, 3 \rangle$$

Method 2. $\vec{u} \times \vec{v} = (\vec{i} - \vec{j} + \vec{k}) \times (2\vec{i} + \vec{j} - 3\vec{k})$

$$= 2\vec{i} \times \vec{i} + \vec{i} \times \vec{j} - 3\vec{i} \times \vec{k}$$

$$-2\vec{j} \times \vec{i} - \vec{j} \times \vec{j} + 3\vec{j} \times \vec{k}$$

$$+2\vec{k} \times \vec{i} + \vec{k} \times \vec{j} - 3\vec{k} \times \vec{k}$$

$$= \vec{0} + \vec{k} + 3\vec{j} + 2\vec{k} - \vec{0} + 3\vec{i} + 2\vec{j} - \vec{i} - \vec{0}$$

$$= 2\vec{i} + 5\vec{j} + 3\vec{k}$$

$$= \langle 2, 5, 3 \rangle$$

b) [2 pts] Find a *unit* vector perpendicular to both $\vec{u}$ and $\vec{v}$

$\vec{u} \times \vec{v} = \langle 2, 5, 3 \rangle$ is perpendicular to both $\vec{u}$ and $\vec{v}$.

So

$$\vec{w} = \frac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|} = \frac{\langle 2, 5, 3 \rangle}{\sqrt{2^2+5^2+3^2}} = \frac{\langle 2, 5, 3 \rangle}{\sqrt{38}} = \left\langle \frac{2}{\sqrt{38}}, \frac{5}{\sqrt{38}}, \frac{3}{\sqrt{38}} \right\rangle$$ is

such a unit vector $\perp$ $\vec{u}$ & $\vec{v}$.

Problem 2 [2 pts] Find an equation of a line parallel to $\vec{v} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ that passes through the point $(0, 3, -1)$. Is the point $(2, 4, 0)$ on this line?

An equation of the line is $\vec{r}(t) = \langle 0, 3, -1 \rangle + t \langle 2, 3, -1 \rangle$.

i.e., $\vec{r}(t) = \langle 2t, 3 + 3t, -1 - t \rangle$

$$\begin{cases} 2 = 2t & \cdots ① \\ \cancel{4} = 3 + 3t & \cdots ② \\ 0 = -1 - t & \cdots ③ \end{cases} \qquad \begin{matrix} ① \Rightarrow t = 1 \\ \\ ③ \Rightarrow t = -1 \end{matrix} \Bigg\} \text{ contradiction.}$$

$\Rightarrow$ system ①, ②, ③ has no solution. $\Rightarrow$ $(2, 4, 0)$ is NOT on this line.

Another way to see this: Let $A(0, 3, -1)$ and $B(2, 4, 0)$ be our points. Then $\vec{AB} = \langle 2-0, 4-3, 0-(-1) \rangle = \langle 2, 1, 1 \rangle$, which is NOT parallel to $\vec{v} = \langle 2, 3, -1 \rangle$. So $(2, 4, 0)$ is not on the line.