Math 1172 - Autumn 2015 Name: _____

Quiz 8 - Take Home Recitation Instructor: _____

SHOW ALL WORK!!! Unsupported answers might not receive full credit. Use the back of this sheet for additional workspace.

Problem 1 [4 pts] Suppose $\vec{u} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{v} = 4\hat{\imath} + 2\hat{\jmath}$.

a) [1 pt] Calculate $\vec{u} \cdot \vec{v}$. $\vec{u} = \langle 3, 4 \rangle$, $\vec{v} = \langle 4, 2 \rangle$

$\vec{u} \cdot \vec{v} = 3 \cdot 4 + 4 \cdot 2 = 12 + 8 = 20 \neq 0$

$\vec{u} \cdot \vec{v} \neq 0 \Rightarrow \vec{u}$ and $\vec{v}$ are not perpendicular.

b) [1 pt] Find $\cos\theta$, where $0 \leq \theta \leq \pi$ is the angle between the vectors $\vec{u}$ and $\vec{v}$. Are $\vec{u}$ and $\vec{v}$ perpendicular?

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$. $|\vec{u}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$, $|\vec{v}| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

$\cos\theta = \dfrac{20}{5 \cdot 2\sqrt{5}} = \dfrac{2}{\sqrt{5}}$. So $\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right)$

c) [2 pts] Find a vector $\vec{P}$ that is parallel to $\vec{v}$ and a vector $\vec{N}$ perpendicular to $\vec{v}$ such that $\vec{u} = \vec{P} + \vec{N}$.

Hint: Draw a picture. What is $proj_{\vec{v}}\vec{u}$?

Let $\vec{P} = \text{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right)\vec{v} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v} = \left(\dfrac{20}{20}\right)\vec{v} = \vec{v}$

$= \langle 4, 2 \rangle$.

Then $\vec{N} = \vec{u} - \vec{P} = \langle 3, 4 \rangle - \langle 4, 2 \rangle = \langle -1, 2 \rangle$

(clearly $\vec{N} \cdot \vec{v} = 0$ and thus $\vec{N} \perp \vec{v}$.)