Math 1172 - Autumn 2015 Name: ____________________

Quiz 7 - In Class Recitation Instructor: ____________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

<u>Problem 1</u> [6 pts] A parametric representation for a certain curve is given by:

$$\begin{cases} x(t) &= 2\sin t \\ y(t) &= 1 - 2\cos t \end{cases}$$

a) [2 pts] Eliminate the parameter to find a description of the curve in terms of x, y.

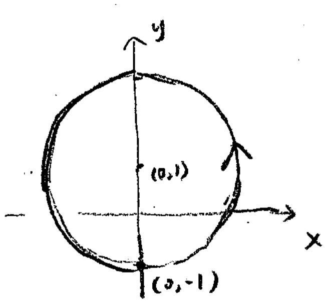
$\frac{x}{2} = \sin t$, $\frac{y-1}{2} = -\cos t$

So $\left(\frac{x}{2}\right)^2 + \left(\frac{y-1}{2}\right)^2 = \sin^2 t + \cos^2 t = 1$

$\Rightarrow x^2 + (y-1)^2 = 4$, a circle centered at $(0,1)$ with radius 2

b) [2 pts] Sketch the curve and indicate the positive orientation.

Take $t=0$ to get a starting point $(2\sin 0, 1-2\cos 0) = (0,-1)$



At t increases from 0, $x(t) = 2\sin t$ increases at first and $y(t) = 1-2\cos t$ increases too at first.

So the positive orientation is counterclockwise.

c) [2 pts] Find the slope of the tangent line to the curve when $t = \frac{\pi}{4}$.

$x'(t) = 2\cos t$, $x'(\frac{\pi}{4}) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2} \neq 0$

$y'(t) = 2\sin t$, $y'(\frac{\pi}{4}) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

The slope of the tangent line is $k = \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \frac{y'(\frac{\pi}{4})}{x'(\frac{\pi}{4})} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

By-product: A point on the line is $(x(\frac{\pi}{4}), y(\frac{\pi}{4})) = (2\sin\frac{\pi}{4}, 1-2\cos\frac{\pi}{4})$
$= (\sqrt{2}, 1-\sqrt{2})$

Hence by the point-slope formula

$y - (1-\sqrt{2}) = 1 \cdot (x - \sqrt{2})$

or $y = x - \sqrt{2} + 1 - \sqrt{2}$, or $y = x + 1 - \sqrt{2}$.