Math 1172 - Autumn 2015 Name: ______________________

Quiz 5 - Take Home Recitation Instructor: ______________________

SHOW ALL WORK!!! Unsupported answers might not receive full credit. CLEARLY label your responses to parts a) and b) and use the back of this paper if necessary.

Problem 1

a) [1 pt] Use partial fraction decomposition to show:

$$\frac{2x^4+4x^2-x+2}{x(x^2+1)^2} = \frac{2}{x} - \frac{1}{(x^2+1)^2}$$

b) [3 pts] Evaluate the integral $\displaystyle\int \frac{2x^4+4x^2-x+2}{x\,(x^2+1)^2}\,dx.$

Hint: Trigonometric substitutions never really go away...

a) $$\frac{2x^4+4x^2-x+2}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

So $2x^4+4x^2-x+2 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$

$= A(x^4+2x^2+1) + (Bx^4+Bx^2+Cx^3+Cx) + (Dx^2+Ex)$

and
$$\begin{cases} A+B=2 & \cdots ① \\ C=0 & \cdots ② \\ 2A+B+D=4 & \cdots ③ \\ C+E=-1 & \cdots ④ \\ A=2 & \cdots ⑤ \end{cases}$$

① & ⑤ $\Rightarrow B=0$

② & ④ $\Rightarrow E=-1$

⑤ & $B=0$ & ③ $\Rightarrow D=0$

i.e., $A=2,\ B=0,\ C=0,\ D=0,\ E=-1$.

Hence, we have the desired decomposition

$$\frac{2x^4+4x^2-x+2}{x(x^2+1)^2} = \frac{2}{x} - \frac{1}{(x^2+1)^2}.$$

See the next page for part (b).

b) By part (a), we have

$$I = \int \frac{2x^4+4x^2-x+2}{x(x^2+1)^2}\,dx = \int \frac{2}{x} - \frac{1}{(x^2+1)^2}\,dx = 2\ln|x| - \int \frac{1}{(x^2+1)^2}\,dx .$$

For the last integral, use trig. sub. $x = \tan\theta$. Then $dx = \sec^2\theta\, d\theta$, $x^2+1 = \tan^2\theta + 1 = \sec^2\theta$, and

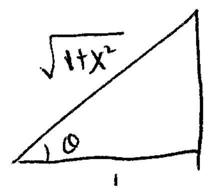
$$\int \frac{1}{(x^2+1)^2}\,dx = \int \frac{\sec^2\theta\, d\theta}{\sec^4\theta} = \int \frac{1}{\sec^2\theta}\,d\theta$$

$$= \int \cos^2\theta\, d\theta = \int \frac{1+\cos(2\theta)}{2}\,d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C$$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cdot\cos\theta + C$$

Ref. triangle for $x = \tan\theta$



$$= \frac{1}{2}\tan^{-1}x + \frac{1}{2}\frac{x}{\sqrt{1+x^2}}\cdot\frac{1}{\sqrt{1+x^2}} + C \quad \leftarrow \quad \sin\theta = \frac{x}{\sqrt{1+x^2}},\ \cos\theta = \frac{1}{\sqrt{1+x^2}}$$

$$= \frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)} + C$$

Hence

$$I = 2\ln|x| - \frac{1}{2}\tan^{-1}x - \frac{x}{2(1+x^2)} + C.$$