Math 1172 - Autumn 2015 Name: _____

Quiz 5 - In Class Recitation Instructor: _____

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [2 pts] Give the general partial fraction decomposition for the following function. DO NOT SOLVE FOR THE CONSTANTS!

$$f(x) = \frac{4x^3 - 7}{x^6 - x^2}$$

Since $x^6 - x^2 = x^2(x^4-1) = x^2(x^2-1)(x^2+1) = x^2(x-1)(x+1)(x^2+1)$,

$$f(x) = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{B}{x-1} + \frac{C}{x+1} + \frac{Dx+E}{x^2+1}$$

Problem 2 [4 pts]

Given that $\frac{2x+36}{(2x-1)(x^2+9)} = \frac{4}{2x-1} - \frac{2x}{x^2+9}$, evaluate:

$$\int_3^\infty \frac{2x+36}{(2x-1)(x^2+9)}\,dx$$

Note: potential "bad" point $x = \frac{1}{2}$ is not in $[3, \infty)$

$$= \lim_{b\to\infty} \int_3^b \frac{2x+36}{(2x-1)(x^2+9)}\,dx = \lim_{b\to\infty} \int_3^b \frac{4}{2x-1} - \frac{2x}{x^2+9}\,dx$$

$$\int \frac{4}{2x-1}\,dx = \frac{4}{2}\ln|2x-1| + C = 2\ln|2x-1| + C = \ln(2x-1)^2 + C$$

$$\int \frac{2x}{x^2+9}\,dx \overset{u = x^2+9}{=\!=} \int \frac{du}{u} = \ln|u| + C = \ln(x^2+9) + C$$

$$= \lim_{b\to\infty} \left[\ln(2x-1)^2 - \ln(x^2+9)\right]\Big|_3^b = \lim_{b\to\infty} \ln\frac{(2x-1)^2}{x^2+9}\Big|_3^b$$

$$= \lim_{b\to\infty} \left(\ln\frac{(2b-1)^2}{b^2+9} - \ln\frac{25}{18}\right)$$

$$= \lim_{b\to\infty} \ln\left(\frac{4b^2-4b+1}{b^2+9}\right) - \ln\left(\frac{25}{18}\right)$$

$$= \ln 4 - \ln\left(\frac{25}{18}\right)$$

$$= \ln\frac{4}{\left(\frac{25}{18}\right)} = \ln\left(\frac{72}{25}\right)$$