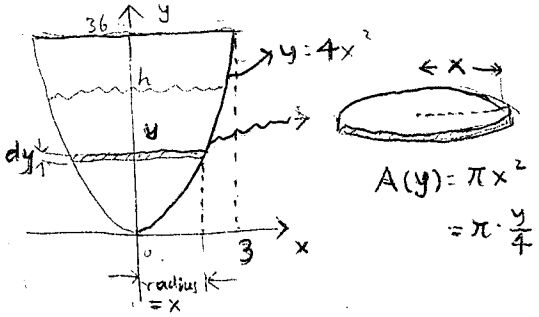
Math 1172 - Autumn 2015 Name: _____

Quiz 3 - Take Home Recitation Instructor: _____

SHOW ALL WORK!!! Unsupported answers might not receive full credit.

Problem 1 [4 points] A tank is formed by revolving the graph of $y = 4x^2$ for $0 \le x \le 3$ (in *meters*) about the y-axis.

a) If the tank is filled with water to the level (height) of h meters, find the volume of the water in terms of h.



$$V = \int_0^h A(y)\,dy = \int_0^h \pi \frac{y}{4}\,dy$$
$$= \frac{\pi}{8} y^2 \Big|_0^h = \frac{\pi h^2}{8}$$

b) If the tank is losing water at the rate of $2\frac{m^3}{s}$, at what rate is the level of the water falling when the level is at 1 meter? (Approximate to 2 decimal places.)

View h as a function of time t. So V is a function of t and $V(t) = \frac{\pi (h(t))^2}{8}$.

Differentiating with respect to t by the chain rule, we get $V'(t) = \frac{\pi}{4} h(t) \cdot h'(t)$

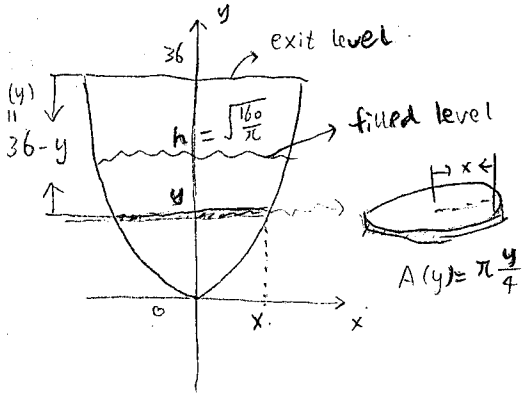
By assumption, $V'(t) = -2$. Suppose at time t_0 we have $h(t_0) = 1$, then

$-2 = \frac{\pi}{4} h(t_0) h'(t_0) = \frac{\pi}{4} h'(t_0)$. Hence, $h'(t_0) = -\frac{8}{\pi}$ m/s. So the desired rate $= \frac{8}{\pi}$ m/s ≈ 2.55 m/s

c) Given that the density of water is $1000\frac{kg}{m^3}$, find the level of the water when there is $20,000\,kg$ of water in the tank. (Approximate to 2 decimal places.)

$$V = \frac{20000}{1000} = 20 = \frac{\pi h^2}{8}. \quad \text{So } h = \sqrt{\frac{20 \cdot 8}{\pi}} = \sqrt{\frac{160}{\pi}} \approx 7.14$$

d) If the tank contains $20,000\,kg$ of water, how much work is done pumping all of the water to an exit pipe at the top of the tank? (Approximate to 2 decimal places.)



$$\text{Work} = \int_0^{\sqrt{160/\pi}} \rho g A(y) \cdot D(y)\,dy$$
$$= \int_0^{\sqrt{160/\pi}} 1000 \cdot 9.8 \cdot \pi \frac{y}{4} (36 - y)\,dy$$
$$= 2450\pi \int_0^{\sqrt{160/\pi}} 36y - y^2\,dy = 245\pi \left(18y^2 - \frac{y^3}{3}\right)\Big|_0^{\sqrt{160/\pi}}$$
$$= 2450\pi \left(18 \cdot \frac{160}{\pi} - \frac{1}{3}\left(\sqrt{\frac{160}{\pi}}\right)^3\right) = 245\pi \left(\frac{2880}{\pi} - \frac{640\sqrt{10}}{3\pi^{3/2}}\right)$$
$$\approx 6123497.80 J$$