AM continued electrical waves

Modulation

\[ v(t) \cos(2\pi f_c t) \]

\[ f_c \gg f_{\text{max}} \]

Mixer

\[ v(t) \cos(2\pi f_c t) \]

\[ \cos(2\pi f_c t) \]

Radio transmitter

Radio frequency

Radio waves (EM waves)

Spectrum of audio signal \( v(t) \)

\[ f_{\text{max}} \approx 3000 \text{ Hz} \]
radio waves (EM waves)

$V_3(t) \cos(2\pi f_{c3} t)$
$V_2(t) \cos(2\pi f_{c2} t)$
$V(t) \cos(2\pi f_c t)$

Radio Receiver

2f_{max} \quad 2f_{max} \quad 2f_{max}

f_{c3} \quad f_{c2} \quad f_c

tuner
band pass filter

Demodulation or Detection
$
\frac{V(t) + dc}{f_c}
$

Diode

C
Periodic Wave forms

\[ x(t + T_0) = x(t) \]

\( T_0 \) is a period

The smallest period is known as the fundamental period

Harmonically related frequencies

Frequencies that are integer multiples of a frequency \( f_0 \)

\[ f_k = k f_0 \] (harmonic frequencies)

\( k = 1 \) 1st Harmonic, \( k = 2 \) 2nd Harmonic etc.

\( f_0 \): fundamental frequency

\( T_0 = \frac{1}{f_0} \) fundamental period

\[ A_k \cos (2\pi k f_0 t + \phi_k) \] \( \forall T_0 \) for all \( k \)
\[ f_0 = \gcd\{f_k\} \]

Greatest Common Divisor

\[
\begin{aligned}
1.2 \text{ Hz, } 2 \text{ Hz, } 6 \text{ Hz}
\end{aligned}
\]

What is the fundamental frequency?

\[ \gcd\{1.2, 2, 6\} = 0.4 \]

1.2 = 0.4 \times 3 \\
2 = 0.4 \times 5 \\
6 = 0.4 \times 15 \\

3rd harmonic \\
51st harmonic \\
15th harmonic
\[ A_k \cos (2\pi k f_0 t + \phi_k) \leq \text{period } T_0 \text{ for all } k \]

\[ x(t) = A_0 + \sum_{k=1}^{N} A_k \cos (2\pi k f_0 t + \phi_k) \]

\[ x(t) \text{ has period } T_0 \quad f_0 = \frac{1}{T_0} \text{ is the fundamental frequency} \]

\[ x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi k f_0 t} = a_0 + 2 \text{Re}\left\{ \sum_{k=1}^{N} a_k e^{j2\pi k f_0 t} \right\} \]

Read section 3-3.1

-2000 Hz -1000 Hz \quad 2000 Hz 2500 Hz 1000 Hz 1500 Hz 2000 Hz

Fundamental frequency 100 Hz

Magnitude spectrum

Phase spectrum

Man speaking the vowel sound "ah"
Now periodic signals

\[ x(t) = A + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) \]

\[ = A + \sum_{k=1}^{N} \left\{ \frac{A_k e^{j\phi_k} e^{j2\pi f_k t} + A_k e^{-j\phi_k} e^{-j2\pi f_k t}}{2} \right\} \]

\[ = \sum_{k=-N}^{N} A_k e^{j2\pi f_k t} \]

If \( f_k \)'s are not harmonically related
then \( x(t) \) is not periodic.
Example

\[ x_h(t) = 2 \cos(20\pi t) - \frac{2}{3} \cos(20\pi (3)t) \]
\[ + \frac{2}{5} \cos(20\pi (5)t) \]

frequencies not harmonically related

\[ f_0 = 10 \text{ Hz} \]

\[ x_h(t) \text{ is periodic with fundamental period } = \frac{1}{f_0} = 0.1 \text{ sec} \]

\[ x_a(t) = 2 \cos(20\pi t) - \frac{2}{3} \cos(20\pi \sqrt{8} t) \]
\[ + \frac{2}{5} \cos(20\pi \sqrt{27} t) \]

frequencies not harmonically related

\[ x_a(t) \text{ is not periodic} \]

See figures 3-10 & 3-11 in the book
Fourier Series \[ x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi / T_0) k t} \]

\[ x(t) \text{ has fundamental period } T_0 \text{ frequency } f_0 = \frac{1}{T_0} \]

Fourier Analysis: given \( x(t) \) determine \( a_k \)

Fourier Synthesis: given \( \{a_k\} \) generate \( x(t) \)

If \( x(t) \) is real then \( a_{-k} = a_k^* \)

\[ x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos \left( \frac{2\pi}{T_0} k t + \phi_k \right) \]

\[ A_0 = a_0 \quad a_k = \frac{1}{2} A_k e^{j\phi_k} \]

By appropriately choosing \( a_k \) we can generate square waves, triangular waves etc.